AKTU

B.Tech Sem-III

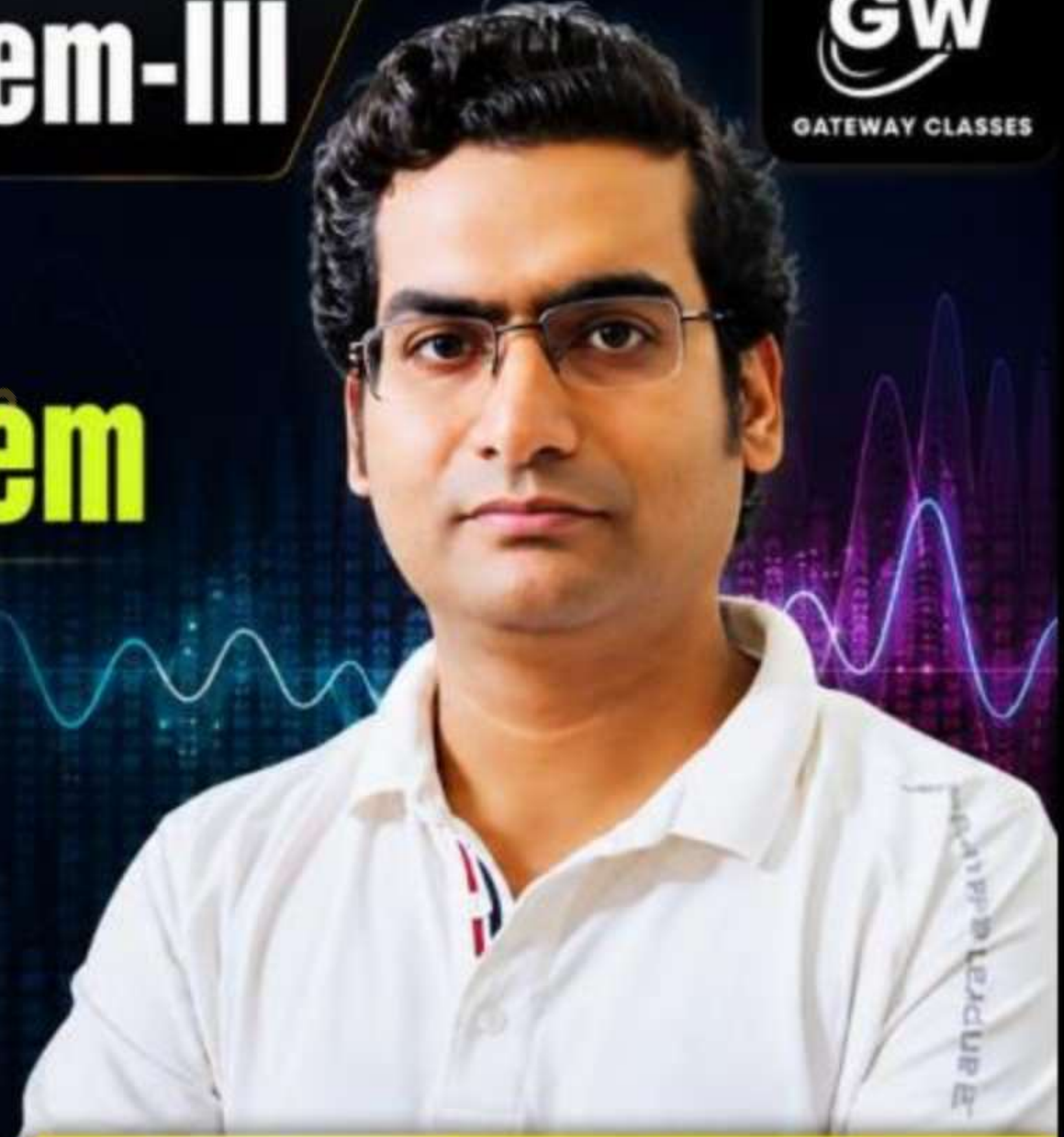
EE and EEE Branch

BSS: Basic Signal System

Unit-1 : Lec-1

Topics to be covered

- Classification of Signals
- Properties of Signals
- Operation of Signals
- AKTU PYQs



By Avinash Sir

GATEWAY CLASSES

Unit -1 Syllabus

Introduction to Continuous Time Signals and Systems: Introduction to continuous time and discrete time signals, Classification of signals with their mathematical representation and characteristics. Transformation of independent variable, Introduction to various type of system, basic system properties.

Analogous System: Linear& Rotational mechanical elements, force-voltage and force-current analogy, modeling of mechanical and electro-mechanical systems; Analysis of first and second order linear systems by classical method.

Classification of Signals

Signal:- Signal is one that carries information and is defined as a physical quantity that varies with one or more independent variable.

Signals are classified into the following categories:

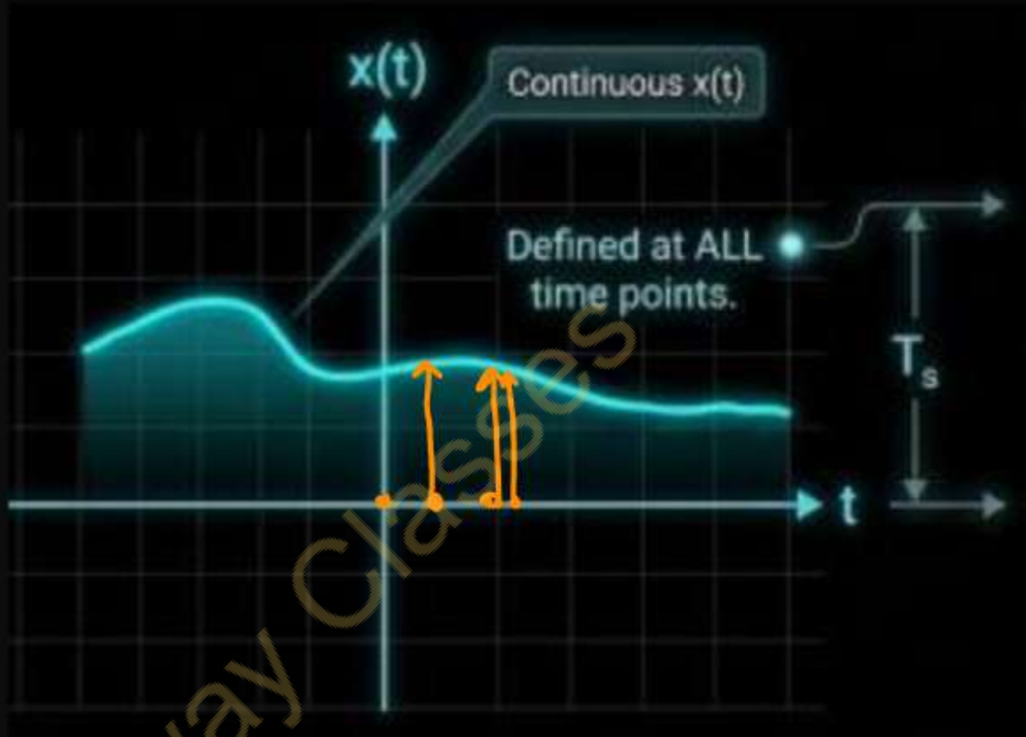
- Continuous Time and Discrete Time Signals
- Even and Odd Signals
- Periodic and Aperiodic Signals
- Energy and Power Signals
- Causal & Non Causal System
- Deterministic and Random signals

Continuous time and discrete time signal

Continuous time signal: A signal that is defined for every instants of time is known as continuous time signal. Continuous time signals are continuous in amplitude and continuous in time. It is denoted by x(t)

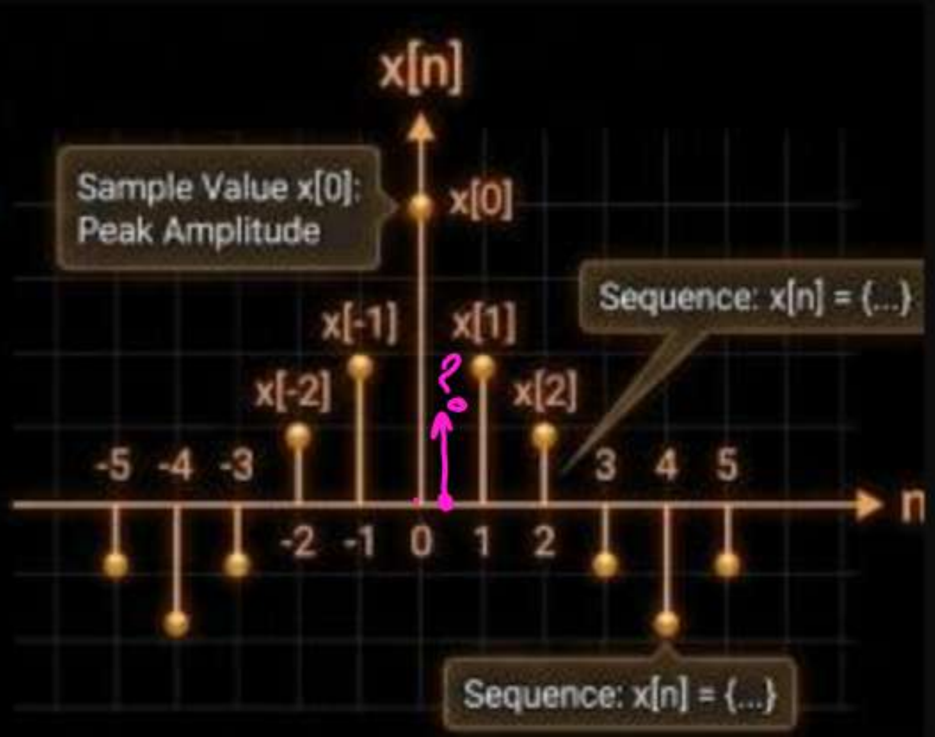
Discrete time signal: A signal that is defined for discrete instants of time is known as discrete time signal. Discrete time signals are continuous in amplitude and discrete in time. It is also obtained by sampling a continuous time signal. It is denoted by x(n) .

$x(t)$


x(t)
Continuous x(t)
Defined at ALL time points.
T_s
t

Graphical representation of continuous-time signal.

$x[n]$


x[n]
Sample Value x[0]: Peak Amplitude
x[0]
x[-1]
x[1]
x[-2]
x[2]
Sequence: x[n] = {...}
-5 -4 -3
3 4 5
-2 -1 0 1 2
n
Sequence: x[n] = {...}

Graphical representation of discrete-time signal.

Even (symmetric) and Odd (Anti-symmetric) signal

Even signal: A signal that exhibits symmetry with respect to t=0 is called even signal Even signal satisfies the condition

$x(t) = x(-t)$

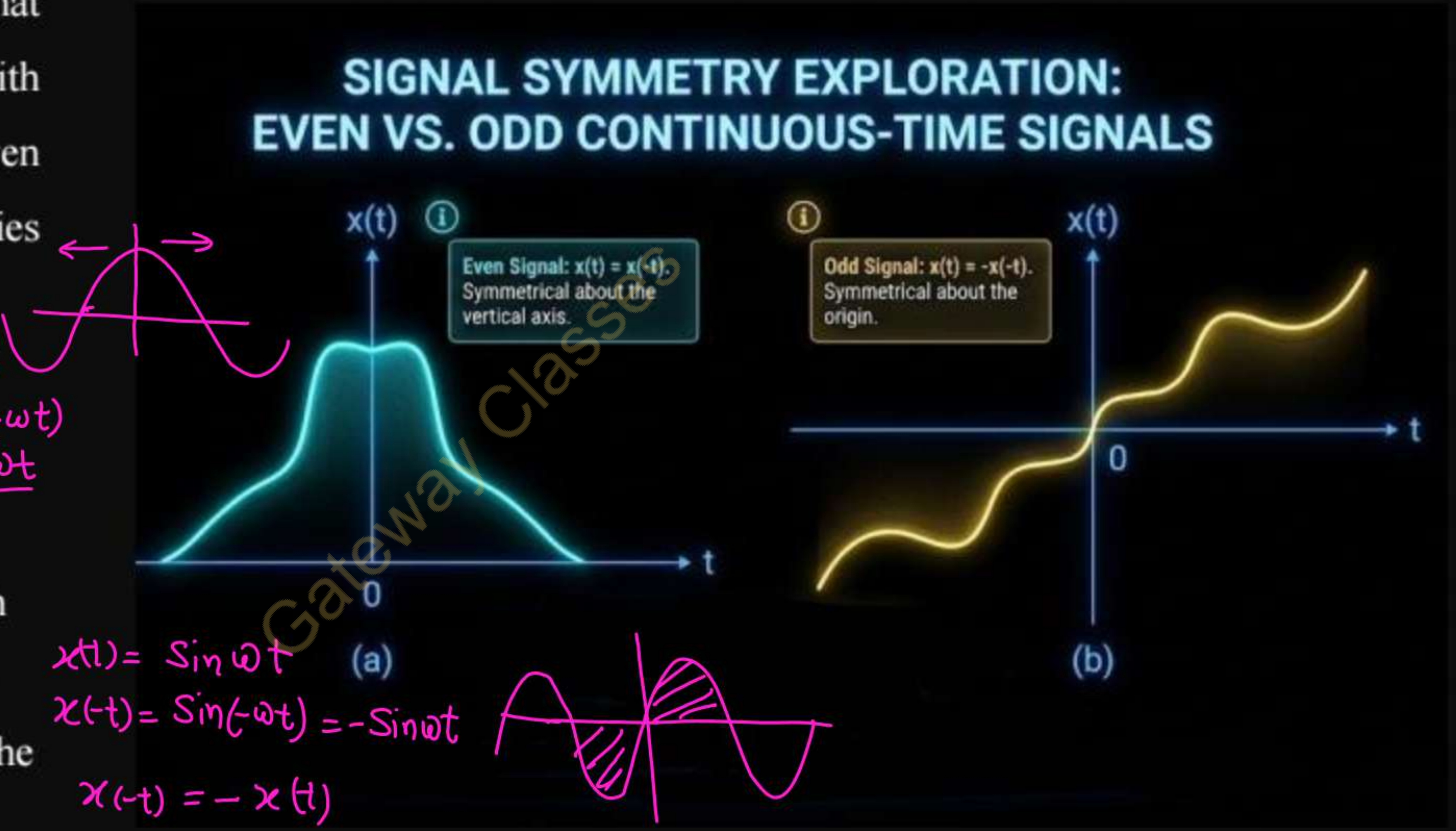
$x(t) = \cos \omega t$

$x(-t) = \cos(-\omega t)$

$= \cos \omega t$

Odd signal: A signal that exhibits anti-symmetry with respect to t=0 is called odd signal Odd signal satisfies the condition

$x(t) = -x(-t)$


SIGNAL SYMMETRY EXPLORATION:
EVEN VS. ODD CONTINUOUS-TIME SIGNALS
x(t)
t
0
(a)
x(t)
t
0
(b)

Even Signal: x(t) = x(-t). Symmetrical about the vertical axis.

Odd Signal: x(t) = -x(-t). Symmetrical about the origin.

$x(t) = \sin \omega t$

$x(-t) = \sin(-\omega t) = -\sin \omega t$

$x(-t) = -x(t)$

Decomposing a Signal into Even and Odd Parts:

Any continuous-time or discrete-time signal can be expressed as the sum of its **even component** and **odd component**.

$$x(t) = \underbrace{x_e(t)}_{\text{Even}} + \underbrace{x_o(t)}_{\text{odd}} \qquad (1.1)$$

where

$x_e(t), x_o(t)$ = Even and odd component of signal $x(t)$

Substituting $t = -t$ in Equation (1.1),

$$x(-t) = x_e(-t) + x_o(-t) \qquad (1.2)$$

Expression for the Even Part $x_e(t)$

For an even signal, $x_e(t) = x_e(-t)$ *(1.3)*

For an odd signal, $x_o(-t) = -x_o(t)$ *(1.4)*

Substituting Equations (1.3) and (1.4) into Equation (1.2),

$$x(-t) = x_e(t) - x_o(t) \qquad (1.5)$$

Adding Equations (1.1) and (1.5),

$$x(t) + x(-t) = [x_e(t) + x_o(t)] + [x_e(t) - x_o(t)]$$

$$x(t) + x(-t) = 2x_e(t)$$

Therefore, $\boxed{x_e(t) = \frac{1}{2}[x(t) + x(-t)]}$ *(1.6)*

Equation (1.6) gives the **even component** of $x(t)$.

Expression for the Odd Part $x_o(t)$

Subtracting Equation (1.5) from Equation (1.1),

$$x(t) - x(-t) = [x_e(t) + x_o(t)] - [x_e(t) - x_o(t)]$$

$$x(t) - x(-t) = 2x_o(t)$$

Therefore, $\boxed{x_o(t) = \frac{1}{2}[x(t) - x(-t)]}$ *(1.7)*

Equation (1.7) gives the **odd component** of $x(t)$.

Examples on Even & Odd Signal

Example 1: $x(t) = e^{-5t}$

$t \to -t$

Step 1: Find $x(-t)$

$$x(t) = e^{-5t}$$

Replacing t by $-t$,

$$x(-t) = e^{-5(\underline{-t})}$$

$$x(-t) = e^{5t}$$

Step 2: Check for Even Signal

For an even signal,

$$x(-t) = x(t)$$

$$e^{5t} \neq e^{-5t}$$

Hence, $x(t)$is **not even**.

Step 3: Check for Odd Signal

For an odd signal,

$$x(-t) = -x(t)$$

$$e^{5t} \neq -e^{-5t}$$

Hence, $x(t)$is **not odd**.

$$\boxed{x(t) = e^{-5t} \textit{ is neither even nor odd}}$$

Example 2: $x(t) = \sin 2\,t$

Step 1: Find $x(-t)$

$$x(t) = \sin 2\,t$$

Replacing $t$by $-t$,

$$x(-t) = \sin 2\,(-t)$$

$$x(-t) = \sin(-2t)$$

Using the identity

$$\sin(-\theta) = -\sin\theta$$

$$x(-t) = -\sin 2\,t$$

$$x(-t) = -x(t)$$

Since

$$x(-t) = -x(t)$$

$$\boxed{x(t) = \sin 2\,t \textit{ is an odd signal}}$$

Examples on Even & Odd Signal

Example 3: $x(t) = \cos 5\,t$

Step 1: Find $x(-t)$

$$x(t) = \cos 5\,t$$

Replacing t by $-t$,

$$x(-t) = \cos 5\,(-t)$$

$$x(-t) = \cos(-5t)$$

Using the identity

$$\cos(-\theta) = \cos\theta$$

$$x(-t) = \cos 5\,t$$

$$x(-t) = x(t)$$

Since

$$x(-t) = x(t)$$

$$\boxed{x(t) = \cos 5\,t \textit{ is an even signal}}$$

Periodic and Aperiodic signal

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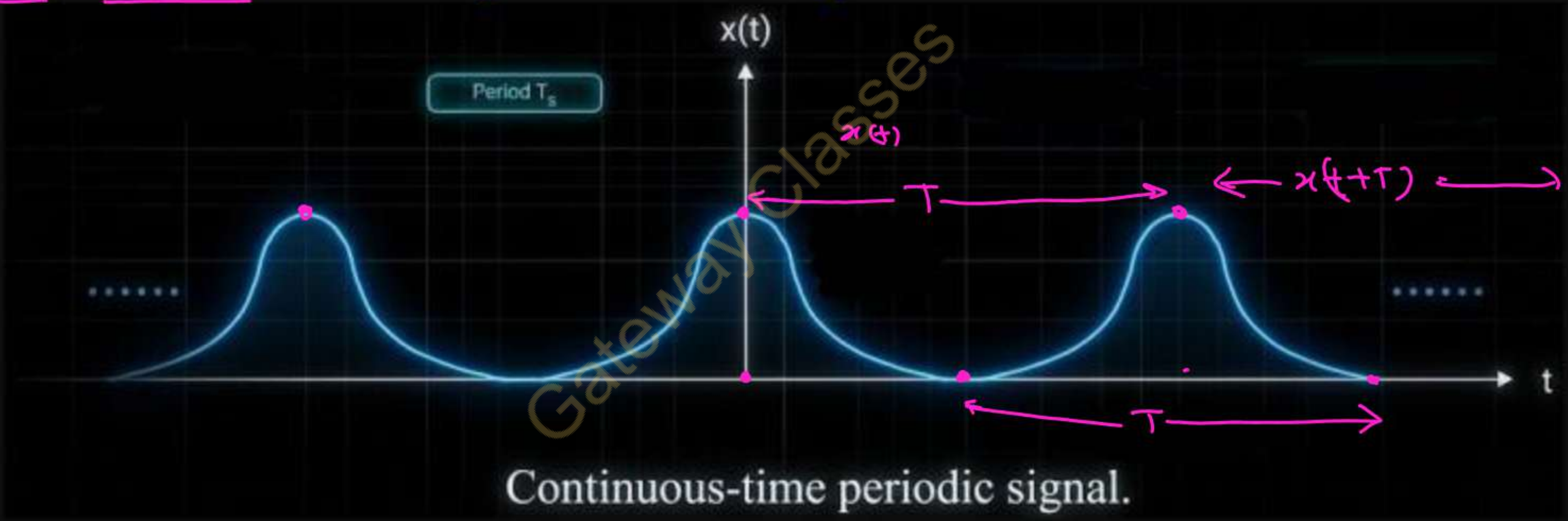
Periodic signal: A signal is said to periodic if it repeats again and again over a certain period of time. Periodic signal should satisfied the condition.

$$x(t) = x(t+T) \text{ where T } \textit{is fundamental time period}$$



Continuous-time periodic signal.

Aperiodic signal: A signal that does not repeat at a definite interval of time is called aperiodic signal.

If $x(t) \neq x(t+T)$ then the signal is said to be aperiodic

Examples on Periodic and Aperiodic signal

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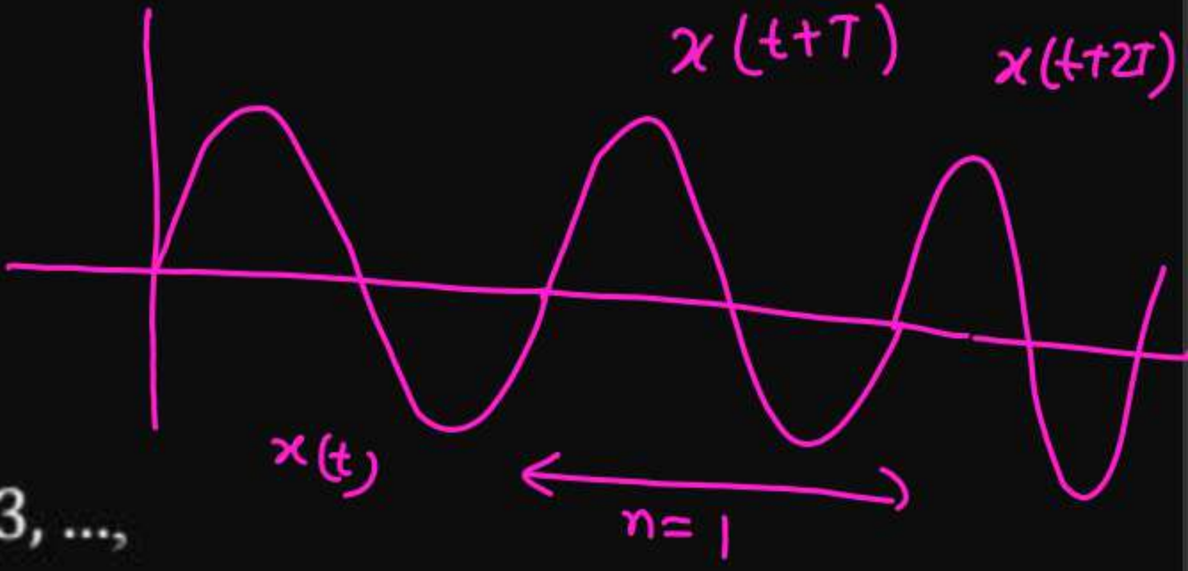
Q. Show that Sinusoidal signal $\underline{x(t) = \sin(\omega t + \theta)}$ is periodic with period $2\pi/\omega$

Condition for Periodicity:- A continuous-time signal is periodic if there exists a positive constant T such that

$$x(\underline{t+T}) = x(t) \qquad \text{for all } t.$$

Step 1: Replace $\underline{t}$ by $\underline{t+T}$



$$x(t+T) = \sin[\omega(t+T)+\theta]$$
$$= \sin(\omega t + \omega T + \theta)$$

Step 2: Apply the Periodicity Condition

For periodicity, $\sin(\omega t + \underline{\omega T} + \theta) = \sin(\omega t + \theta)$

Since

$$\sin(\alpha + \underline{2n\pi}) = \sin\alpha\,, \qquad \text{where } n = 1,2,3,\ldots,$$

we require

$$\omega T = 2n\pi$$
$$T = \frac{2n\pi}{\omega}$$

Step 3: For Fundamental Time Period, $n = 1$.

$$\boxed{T = \frac{2\pi}{\omega}}$$

is the fundamental period.

Examples on Periodic and Aperiodic signal

Example 1: $x(t) = \cos(10\pi t)$

Step 1: Compare with the standard form

The standard periodic signal is

$$x(t) = \cos(\omega t)$$
$$\omega = \underline{10\pi}\ rad/s$$

Step 2: Calculate the fundamental period

For a cosine signal,

$$\boxed{T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = \frac{1}{5}\ s}$$

Proved

Verification

$$x(t+T) = \cos\left[10\pi\left(t + \frac{1}{5}\right)\right]$$
$$= \cos(10\pi t + 2\pi)$$

Since

$$\cos(\theta + 2\pi) = \cos\theta$$
$$x(t+T) = \cos(10\pi t) = x(t)$$
$$\boxed{x(t) = \cos(10\pi t)\ is\ periodic}$$

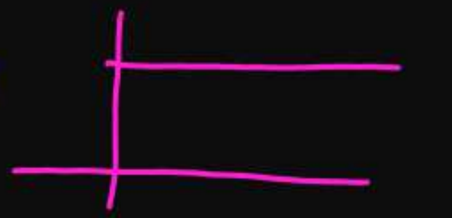
with

$$\boxed{T = \frac{1}{5}\ s}$$

Example 2: $x(t) = \sin(\pi t)\, u(t)$

$\omega = \pi$

Step 1: Examine individual components



Sinusoidal part $= \sin(\pi t)$

has period $T = \frac{2\pi}{\pi} = 2$, Thus, $\sin(\pi t)$ is periodic.

Unit Step Function Part

The unit step function does not repeat itself after any finite interval. Hence, $u(t)$ is **aperiodic**.

Step 2: Product of periodic and aperiodic signals

$$x(t) = \sin(\pi t)\, u(t)$$

Since the unit step function destroys the repeating nature of the sine wave, $x(t+T) \neq x(t)$ for all positive finite T.

$$\boxed{x(t) = \sin(\pi t)\, u(t)\ is\ aperiodic}$$

Combination of Periodic Signals

Ex.

$x(t) = \underline{5\sin(100\pi t)} + 10\cos(10t) + 50\sin\left(20t + \frac{\pi}{2}\right)$

Let the three signals have periods T_1, T_2, and T_3.

Step 1: Determine Individual Time Periods

For First Signal $x_1(t) = 5\sin(100\pi t)$

Angular frequency: $\omega_1 = 100\pi$

Period: $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{100\pi} = \frac{1}{50}\ s$

Second Signal $x_2(t) = 10\cos(10t)$

$\omega_2 = 10$

Period: $T_2 = \frac{2\pi}{10} = \frac{\pi}{5}\ s$

Third Signal $x_3(t) = 50\sin\left(\underline{20}t + \frac{\pi}{2}\right)$

$\omega_3 = 20$

$T_3 = \frac{2\pi}{20} = \frac{\pi}{10}\ s$

Step 2: For a sum of periodic signals to be periodic, the ratios of their periods must be rational numbers.

$\frac{T_2}{T_3} = \frac{\pi/5}{\pi/10} = 2$ (Rational) ✓,

$\frac{T_1}{T_2} = \frac{1/50}{\pi/5} = \frac{1}{10\pi}$ (Since π is irrational, $\frac{1}{10\pi}$ is ✓ irrational.)

$\frac{T_1}{T_3} = \frac{1/50}{\pi/10} = \frac{1}{5\pi}$ (irrational)

Step 3: Since all period ratios are **not rational**, a common fundamental period does not exist.

Therefore,

$$\boxed{x(t) = 5\sin(100\pi t) + 10\cos(10t) + 50\sin\left(20t + \frac{\pi}{2}\right)}$$

is an $\boxed{\text{Aperiodic } Signal}$

Combination of Periodic Signals

(ii) $x(t) = 3\cos(2\pi t) + 5\cos(7t$ [redacted] $)$

$3\cos \underbrace{2\pi}t$

$\omega_1 = 2\pi$

$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{2\pi} = 1$

$5\cos \underbrace{7}t$

$\omega = 7$

$T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{7}$

Ratio $\frac{T_1}{T_2} = \frac{1}{2\pi/7} = \frac{7}{2\pi}$ (irrational)

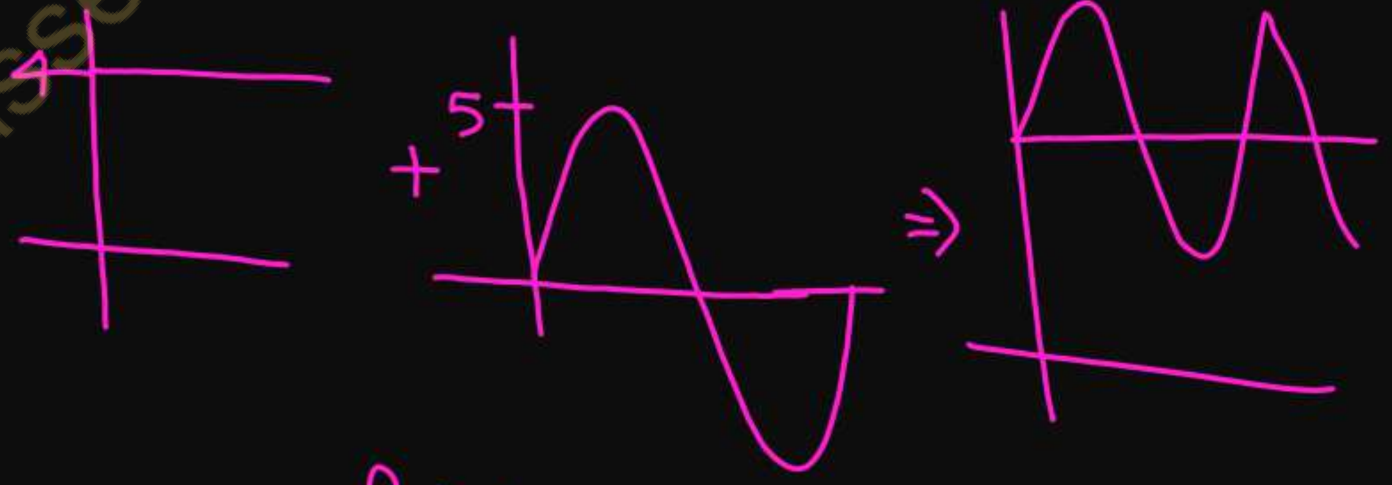
Aperiodic Signal

(ii) $x(t) = \underline{4} + 5\sin(100\pi t)$

$5\sin \underbrace{100\pi}t$

$\omega = 100\pi$

$T = \frac{\cancel{2\pi}}{100\cancel{\pi}} = \frac{1}{50}$



Periodic

Energy and Power signal

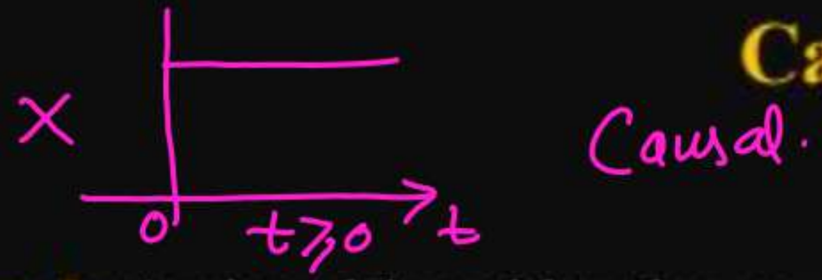
Energy signal: The signal which has finite energy and zero average power is called energy signal. The non periodic signals like exponential signals will have constant energy and so non periodic signals are energy signals. i.e., **For energy signal, $\underline{0} < \underline{E} < \underline{\infty}$ *and* $P_{avg} = 0$**

For Continuous time signals, **Energy** $E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt$

Power signal: The signal which has finite average power and infinite energy is called power signal. The periodic signals like sinusoidal complex exponential signals will have constant power and so periodic signals are power signals. i.e., For power signal, $\underline{0} < P < \underline{\infty}$ *and* $E = \underline{\infty}$

For Continuous time signals

$$\textbf{Average Power} \quad P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$



Causal and Non-causal signal

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Causal signal: A signal is said to be causal if it is **defined for t≥0**.

$$i.e., x(t) = 0 \ for \ t < 0$$

defined only for +ve value (future value)

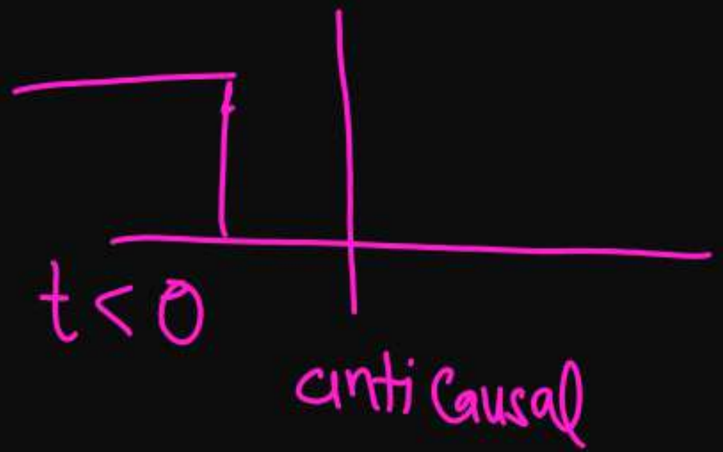
$u(t)$, $u(t-a)$

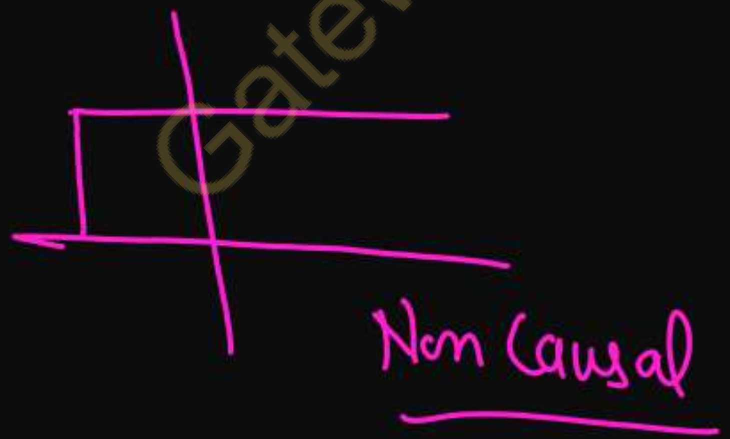
Non-causal signal: A signal is said to be non-causal, if it is **defined for t< 0 or for both $t < 0$ and $t \geq 0$**

$$i.e., x(t) \neq 0 \ for \ t < 0$$

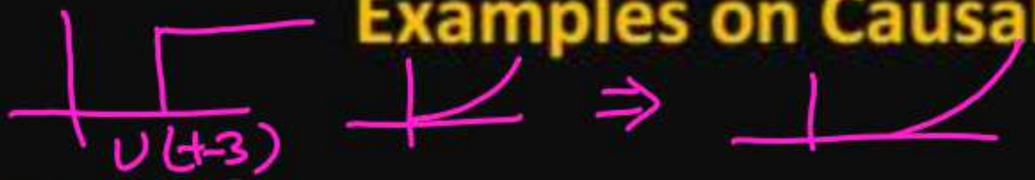
$\underline{u(t+a)}$

When a non-causal signal is defined only for t<0, it is called as **anti-causal signal**





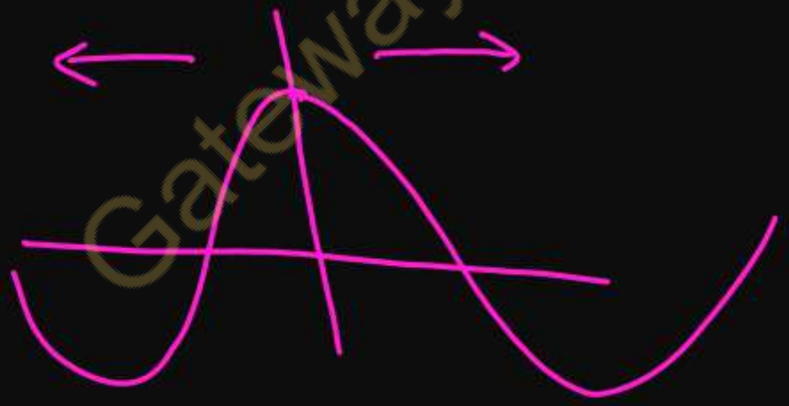
Examples on Causal and Non Causal Signals



1. $x(t) = e^t u(t-3)$ Signal is Causal as it is not defined for $t<3$

2. $x(t) = \sin 3t\ u(t)$ Signal is Causal as it is not defined for $t<0$

3. $x(t) = 3.5\ u(-t)$ Signal is Non Causal as it is defined for $t<0$



4. $x(t) = \cos 3t$ Non Causal

5. $x(n) = u(n+3) - u(n-3)$ Non Causal

(a) A signal $x(t)= e^{-2t}u(t) + e^{2t}u(-t)$ (i) Test whether the signal is energy or power signal (ii) Find its total energy (iii) Comment on its causality

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(i) Step 1: Rewrite Signal

$$x(t) = \begin{cases} e^{2t}, & t < 0 \\ e^{-2t}, & t \geq 0 \end{cases}$$

or equivalently, $\boxed{x(t) = e^{-2|t|}}$

(i) Test Whether the Signal is an Energy Signal or Power Signal

Energy $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$

Substituting $x(t)$,

$$E = \int_{-\infty}^{0} (e^{2t})^2 dt + \int_{0}^{\infty} (e^{-2t})^2 dt$$

$$E = \int_{-\infty}^{0} e^{4t}\, dt + \int_{0}^{\infty} e^{-4t}\, dt$$

$$\mathbf{E} = \left[\frac{e^{4t}}{4}\right]_{-\infty}^{0} + \left[-\frac{e^{-4t}}{4}\right]_{0}^{\infty} = \frac{1}{4} + \frac{1}{4} = \boxed{E = \frac{1}{2}}$$

Since the energy is finite,

$$0 < E < \infty$$

the signal is an **Energy Signal**.

Average Power $P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\, dt$

Since the total energy is finite,

$$\boxed{P = 0}$$

$$\boxed{\text{The } \textit{signal is an Energy Signal.}}$$

(ii) Total Energy $\boxed{E = \frac{1}{2}\ units}$ ✓

(iii) Comment on its Causality

A signal is **causal (right-sided)** if $x(t) = 0, t < 0$

For the given signal, $x(t) = e^{2t}, t < 0$

which is **non-zero** for negative time.

Therefore, $\boxed{x(t) \neq 0\ for\ t < 0}$

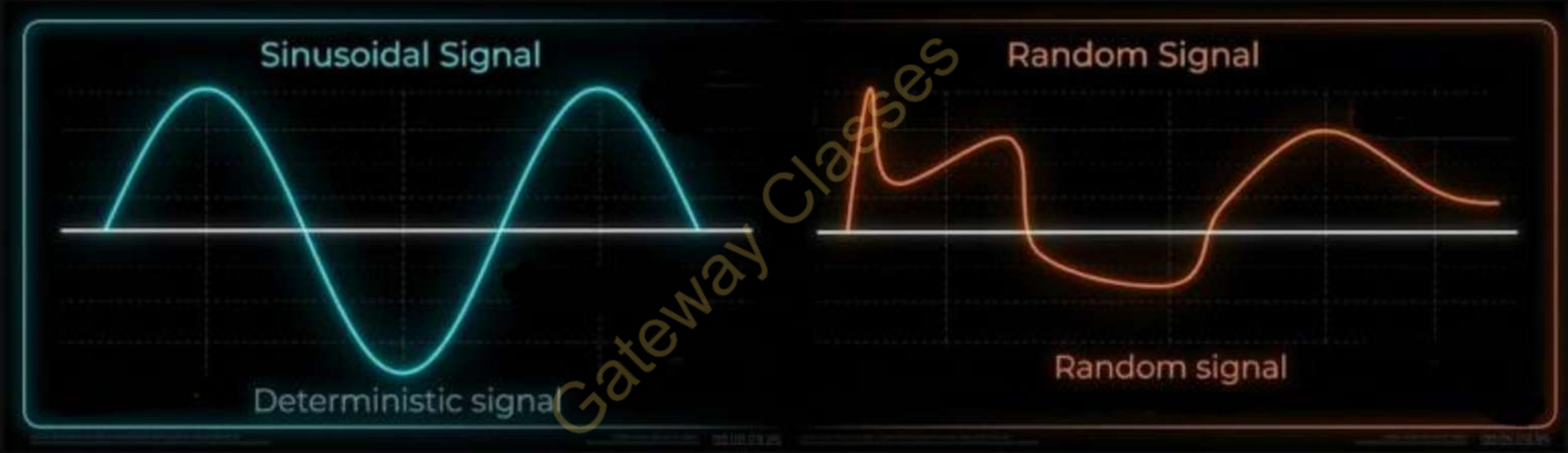
Hence the signal exists before $t = 0$.

$$\boxed{\text{The } \textit{signal is Non−Causal (two−sided).}}$$

Deterministic and Random signals

Deterministic signal: A signal is said to be deterministic if there is no uncertainty over the signal at any instant of time i.e., its instantaneous value can be predicted. It can be represented by mathematical equation.

Example: sinusoidal signal


Sinusoidal Signal
Deterministic signal
Random Signal
Random signal

Random signal (Non-Deterministic signal): A signal is said to be random if there is uncertainty over the signal at any instant of time i.e., its instantaneous value cannot be predicted. It cannot be represented by mathematical equation. Example: noise signal

Basic(Elementary or Standard) continuous time signals

Step signal:-

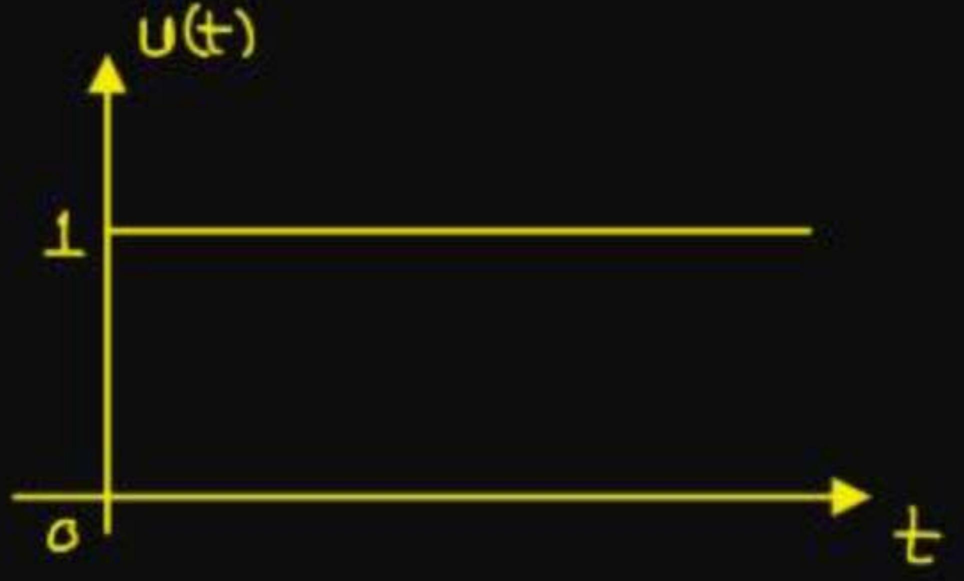
Unit Step signal is defined as

$$u(t) = 1 \quad for \quad t \geq 0$$
$$= 0 \quad for \quad t < 0$$

step signal

$\underline{A}$ for $t \geq 0$

0 for $t < 0$



$$u(t) \xrightarrow{\int} r(t) = t\,u(t) \xrightarrow{\int} \text{Parabolic}$$

$$\frac{t^2}{2}u(t)$$

Ramp signal

$$\boxed{r(t) = t \cdot u(t)}$$

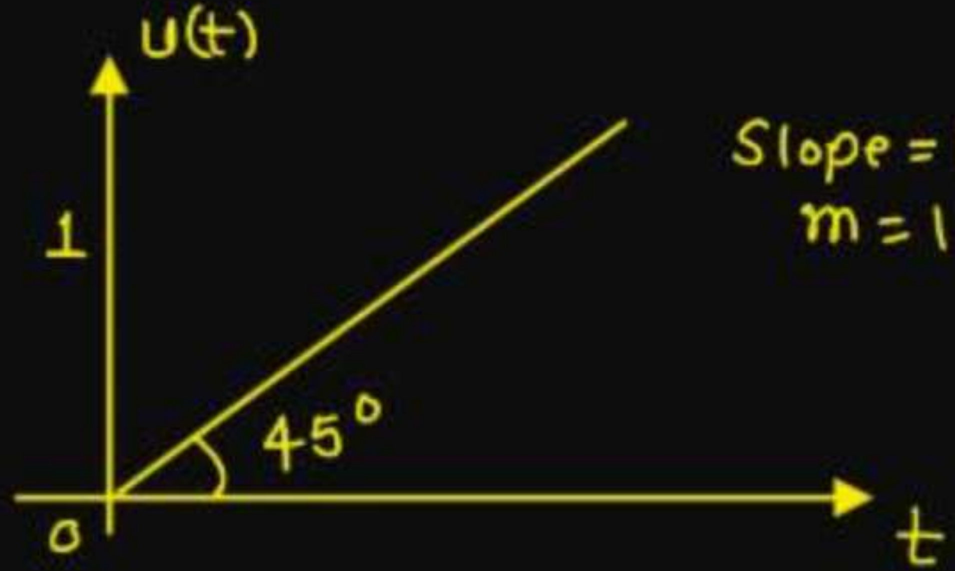
Unit ramp signal is defined as

$$r(t) = t \quad for \quad t \geq 0$$
$$= 0 \quad for \quad t < 0$$

$r(t) = m\,t$

$\tan\theta = 1$

$$\boxed{\theta = 45^\circ}$$



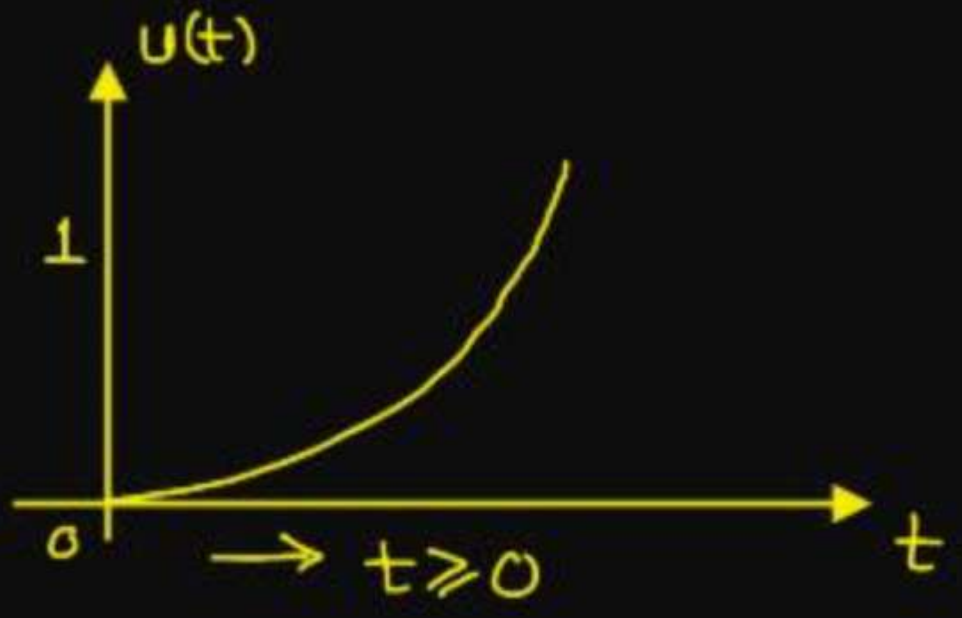
Slope = 1

m = 1

Basic(Elementary or Standard) continuous time signals

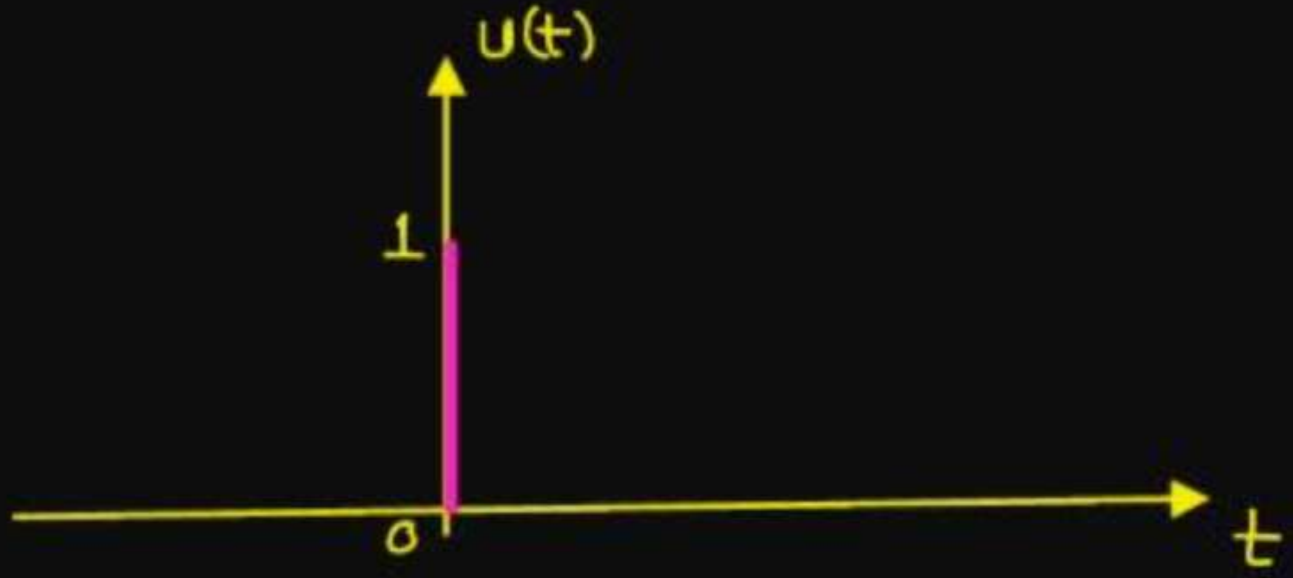
Parabolic signal:- Unit Parabolic signal is defined as

$$P(t) = \begin{cases} \frac{t^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$



Impulse signal:- Unit Impulse signal is defined as

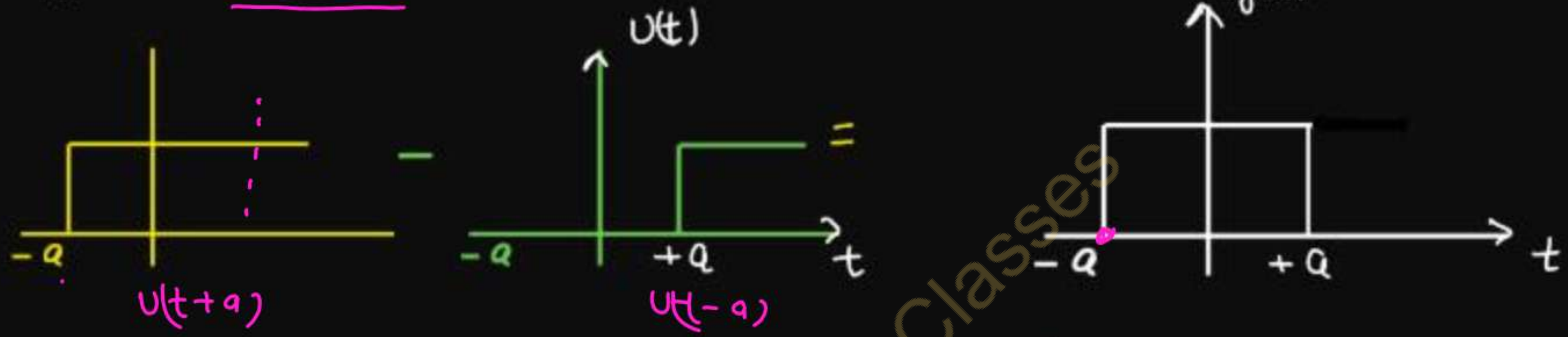
$$\underline{\delta(t)} = x(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$



Basic(Elementary or Standard) continuous time signals

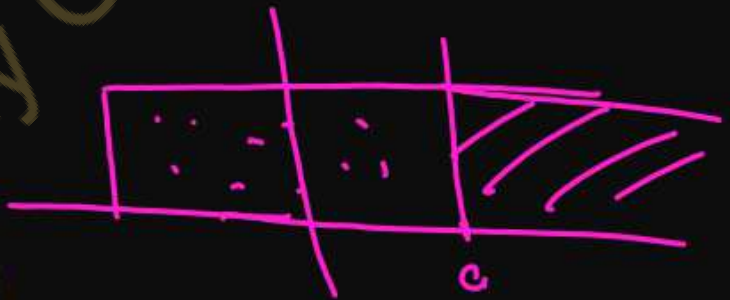
Gate Function:- Gate function is given by,

$$G(t) = \underline{u(t+a)} - u(t-a)$$



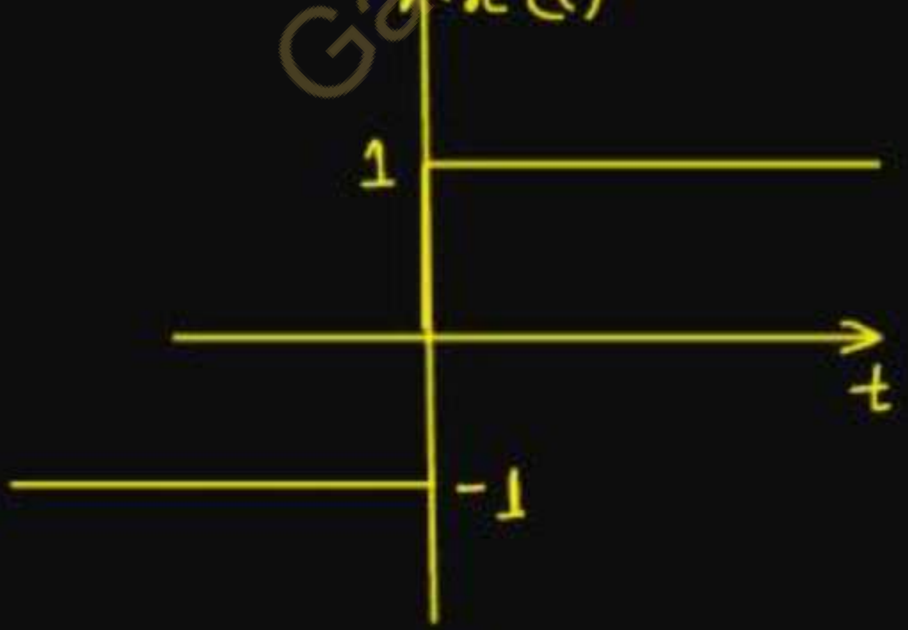
-a
u(t+a)
-
u(t)
-a
+a
t
u(t-a)
=
g(t)
-a
+a
t

Signum Function:- Signum function is defined as



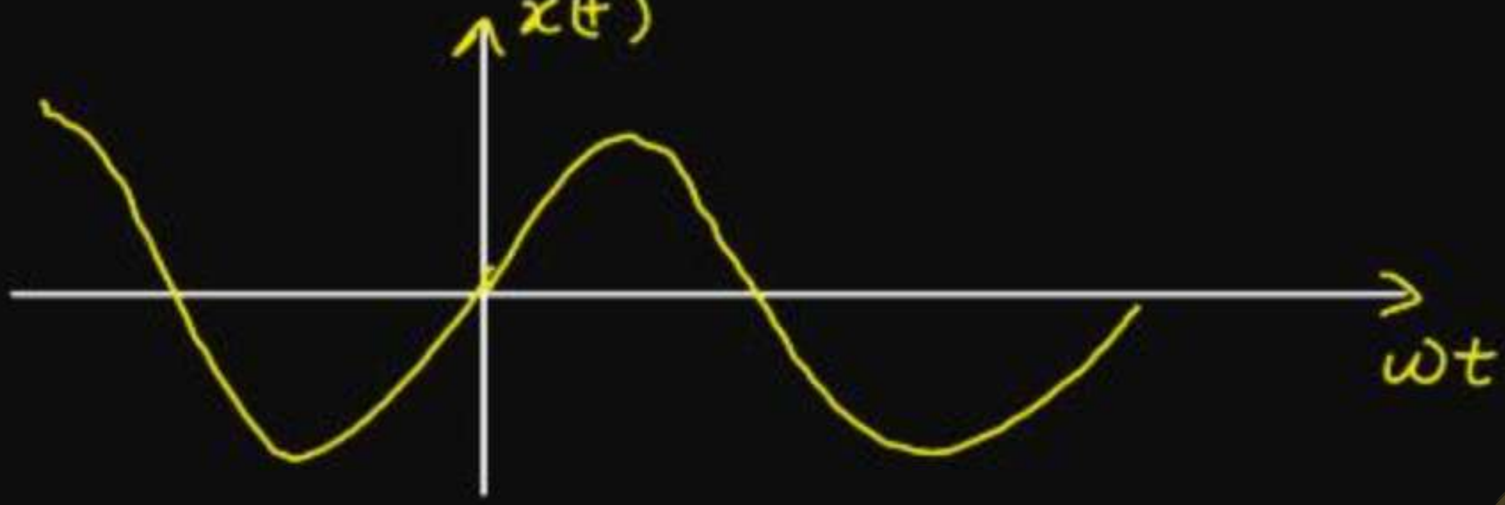
$$Sgn(t) = u(t) - u(-t)$$

$$Sgn(t) = \begin{cases} 1 & t>0 \\ 0 & t=0 \\ -1 & t>0 \end{cases}$$



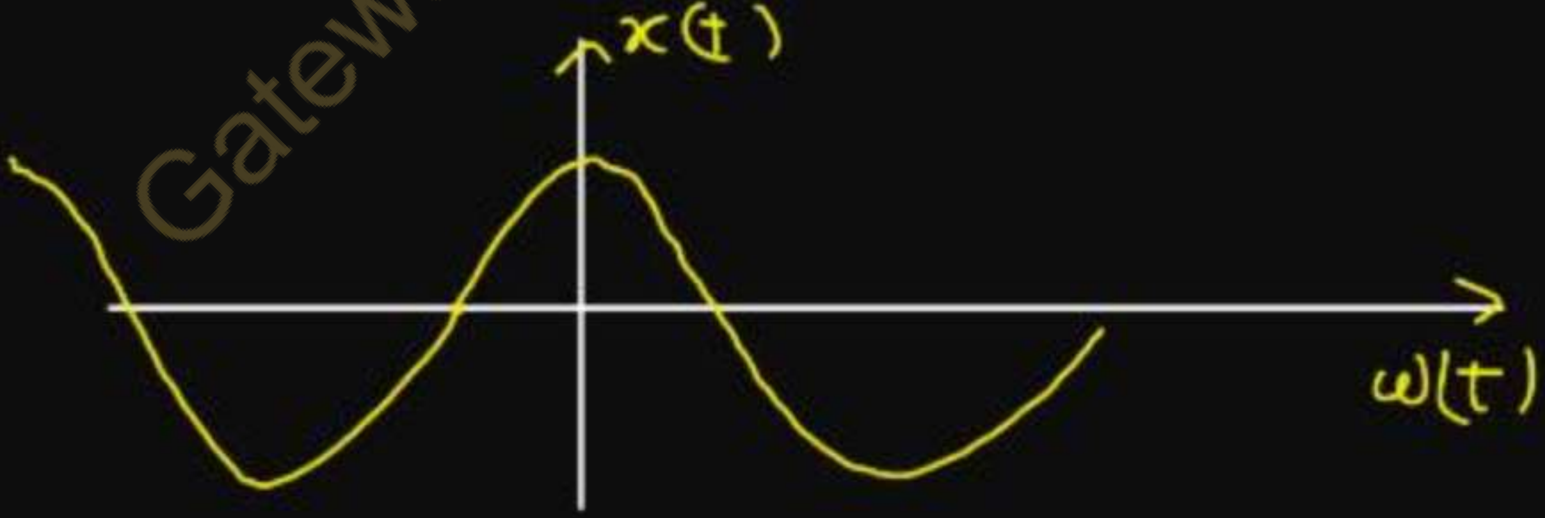
Basic(Elementary or Standard) continuous time signals

Sinusoidal signal:- Given by $x(t) = A\sin(\omega t)$



Co-sine signal:- Given by $x(t) = A\cos(\omega t)$

Where, A= Amplitude of wave
ω= angular frequency rad/sec,
Φ= phase shift in rad.



Basic(Elementary or Standard) continuous time signals

Exponential signal

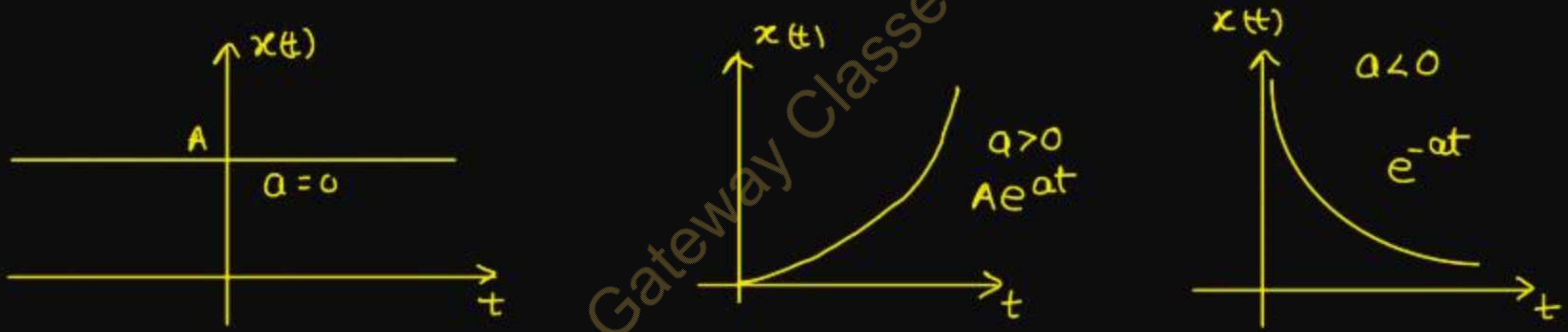
Ae^{-at}

Real Exponential signal is defined as $x(t) = Ae^{at}$

$a = 0$

where A is amplitude

Depending on the value of ‘a’ we get dc signal or growing exponential signal or decaying


x(t)
A
a = 0
t
x(t)
a > 0
Ae^{at}
t
x(t)
a < 0
e^{-at}
t

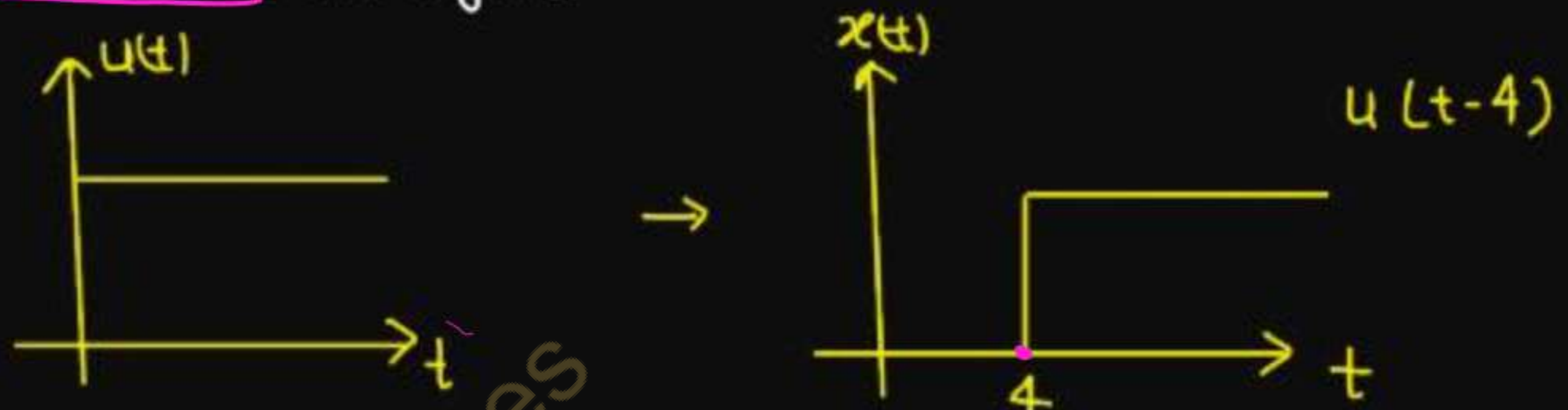
Operation on Elementary Signals (Properties)

Time Shifting:- It generate delay or advances in signal

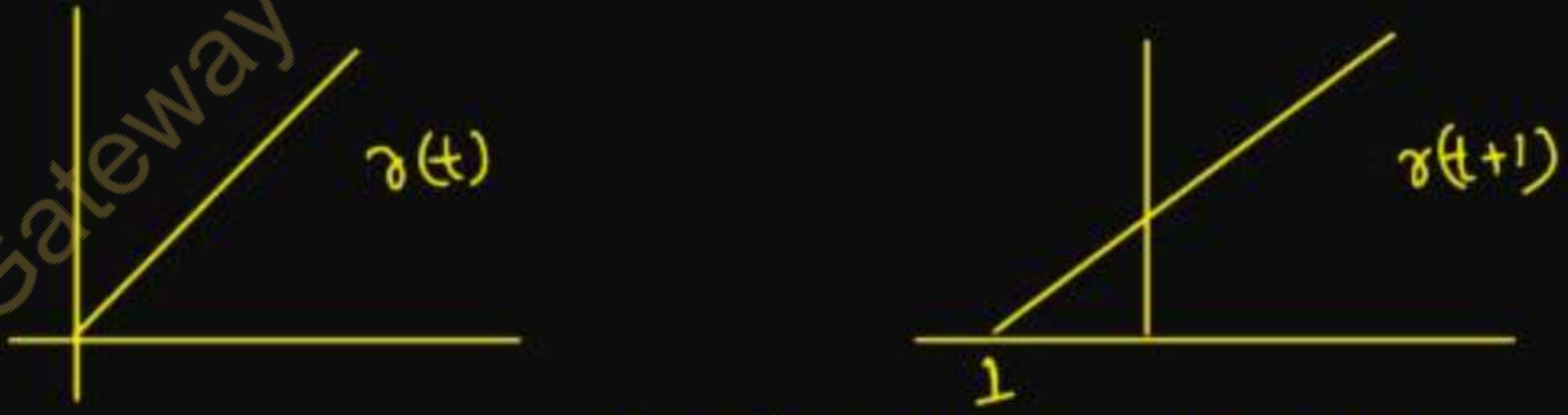
$x(t \pm t_0)$

$+ \rightarrow$ Advance

$- \rightarrow$ delay



Signal shift to right side



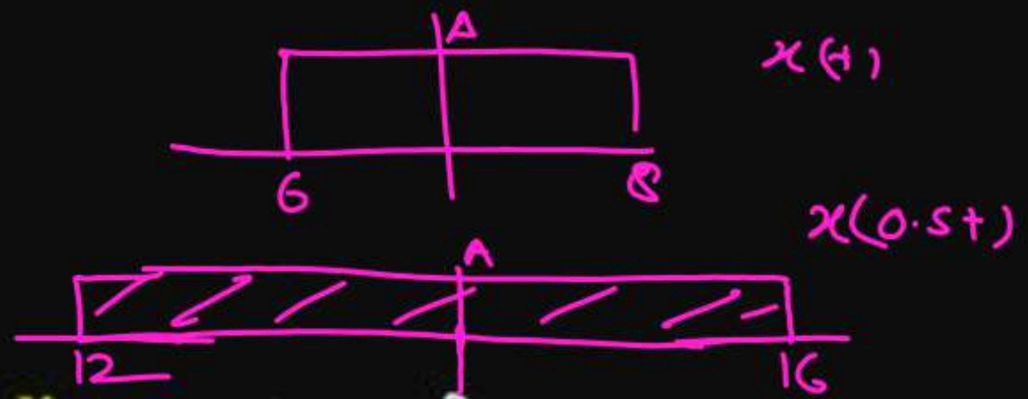
Signal shift to left side

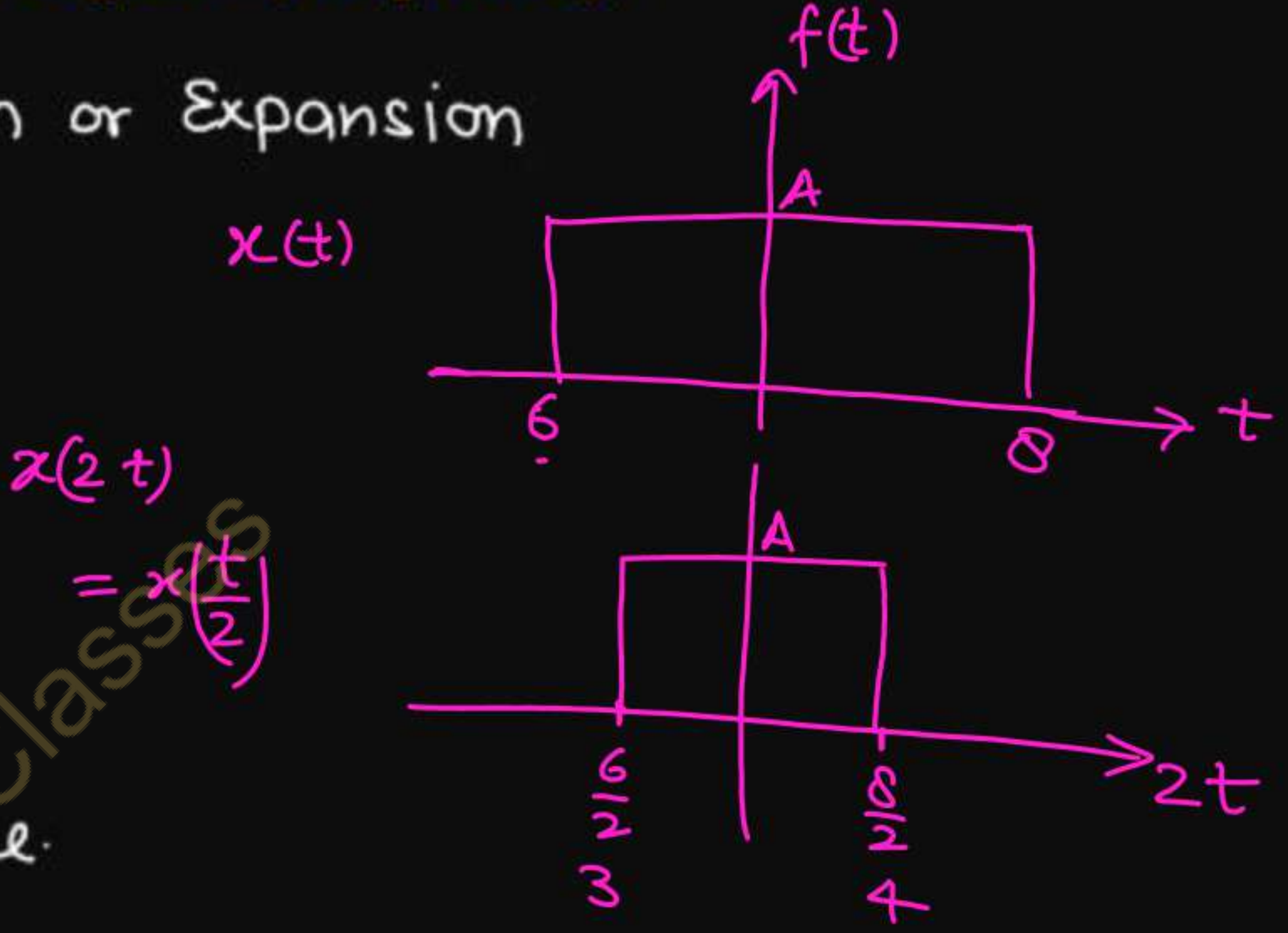
Operation on Elementary Signals (Properties)

Time Scaling:- It is used for signal Compression or Expansion

$x(at)$ if $a>1 \longrightarrow$ Compression

$a<1 \longrightarrow$ Expansion

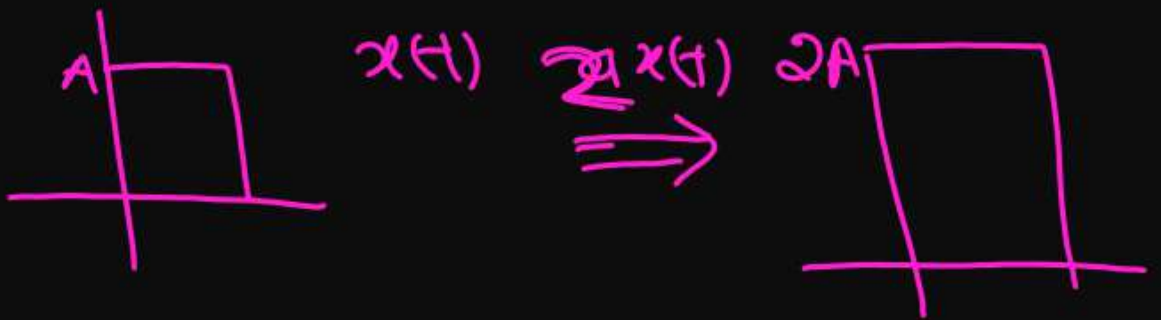

A
x(t)
6
8
x(0.5t)
A
12
16


f(t)
A
x(t)
6
8
t
x(2t)
= x(t/2)
A
6/2
3
8/2
4
2t

Amplitude Scaling:- Used for increasing Amplitude.

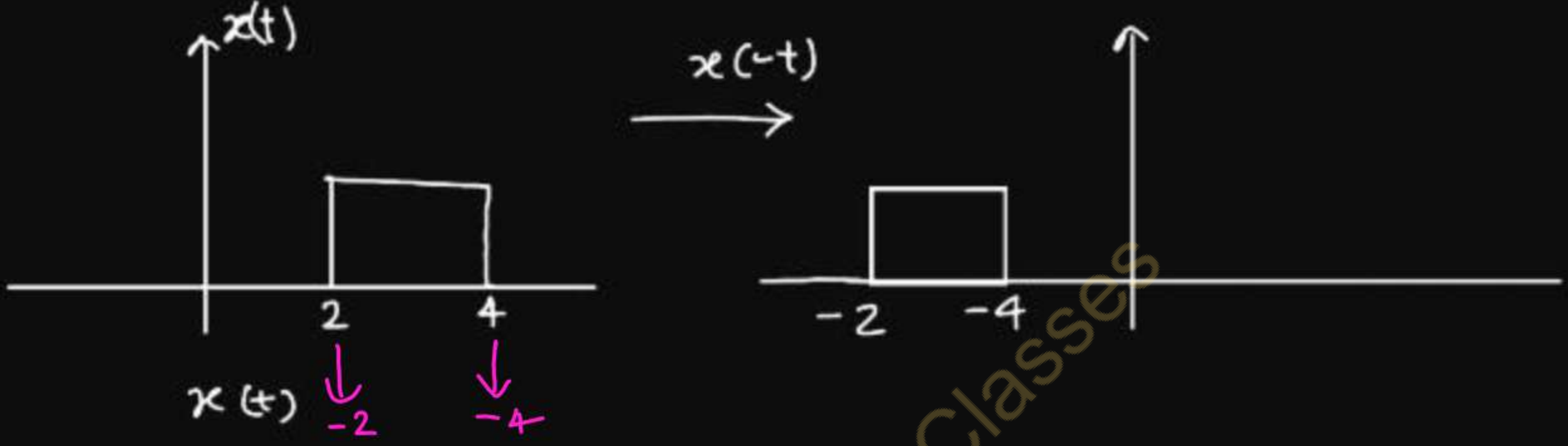
if $a>1$ Amplitude increase

$a<1$ Amplitude decrease



Operation on Elementary Signals (Properties)

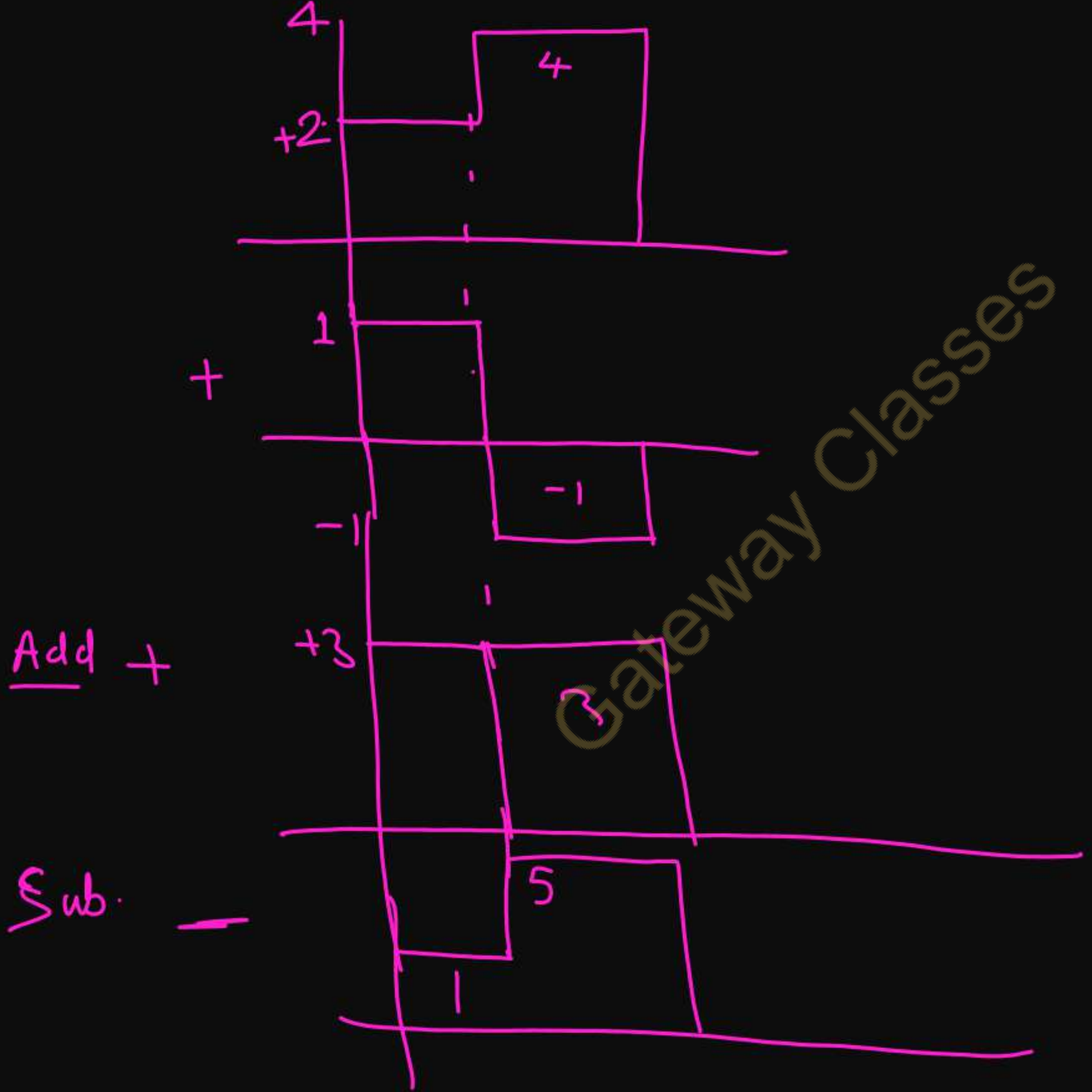
Time Reversal (Folding):- This property fold Signal along vertical axis



x(t)
2
4
x(t)
-2
-4
x(-t)
-2
-4

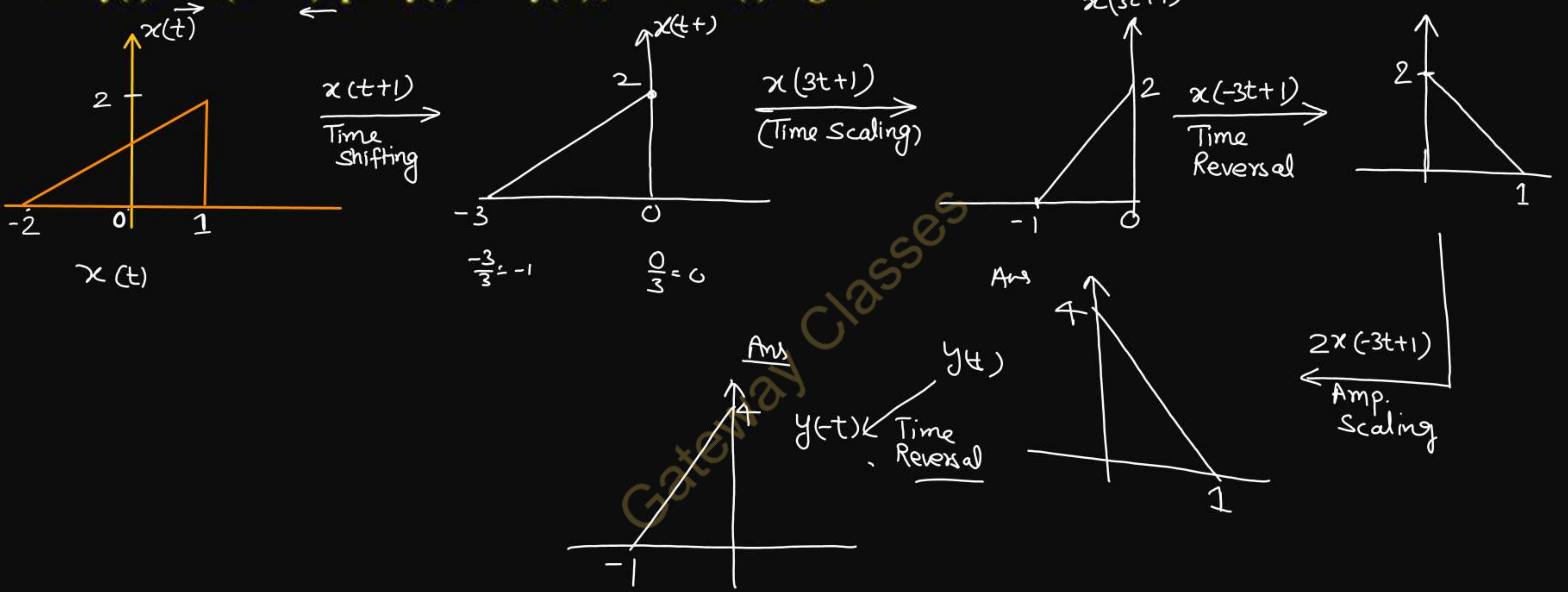
Operation on Elementary Signals (Properties)

Addition or subtraction:-


4
+2
4
1
+
-1
-1
Add +
+3
3
Sub. –
5
1

Operation on Elementary Signals (Properties)

Ex.1:- y(t) = 2x(-3t+1) plot y(t) and y(-t) , where x(t) is given as



x(t)
2
-2
0
1
x (t)
x(t+1)
Time shifting
x(t+)
2
-3
0
-3/3 = -1
0/3 = 0
x(3t+1)
(Time Scaling)
x(3t+1)
2
-1
0
Ans
x(-3t+1)
Time Reversal
2
1
2x(-3t+1)
Amp. Scaling
4
1
y(t)
y(-t)
Time Reversal
Ans
4
-1

Operation on Elementary Signals (Properties)

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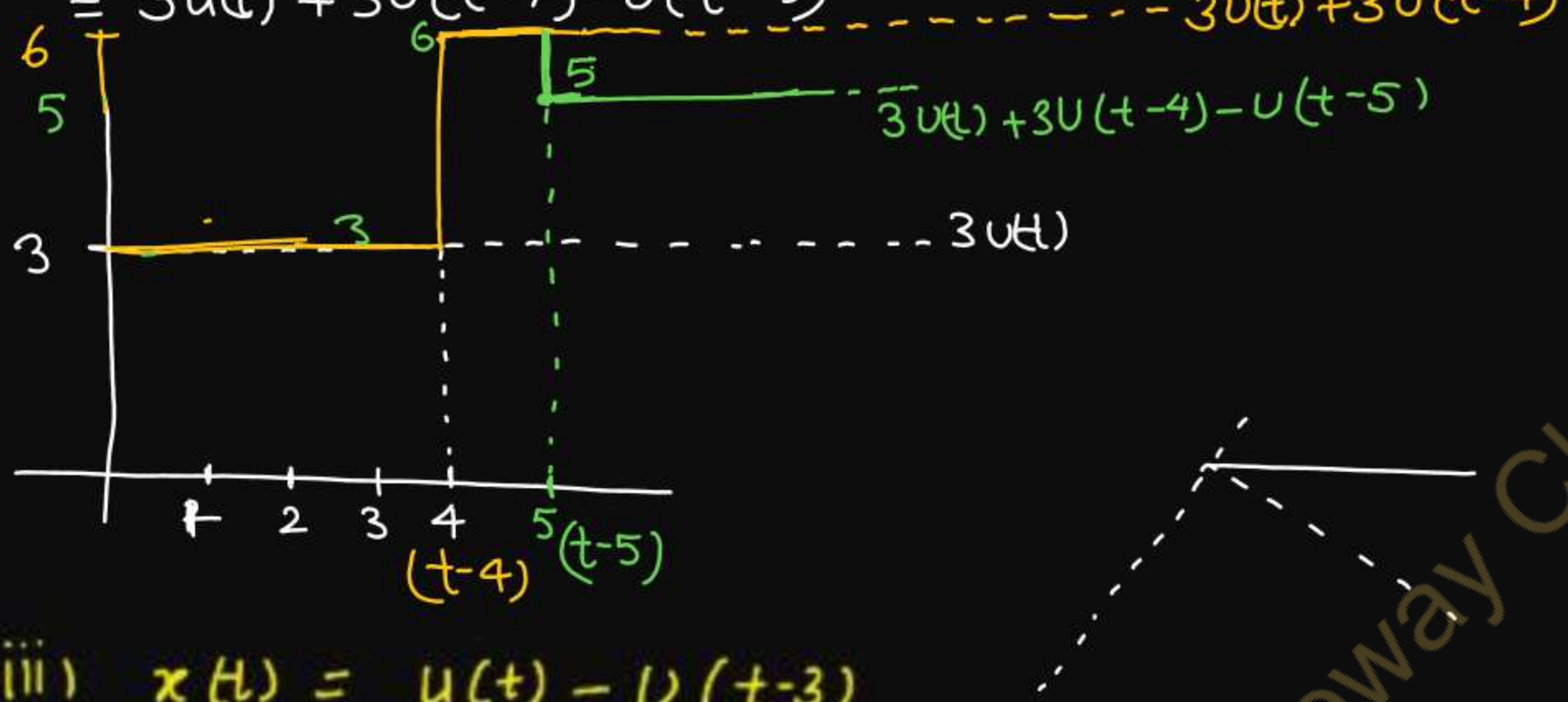
Observe the function graphically

$x(t)=u(t) + 2u(t) + 3u(t-4) - u(t-5)$

$= 3u(t) + 3u(t-4) - u(t-5)$

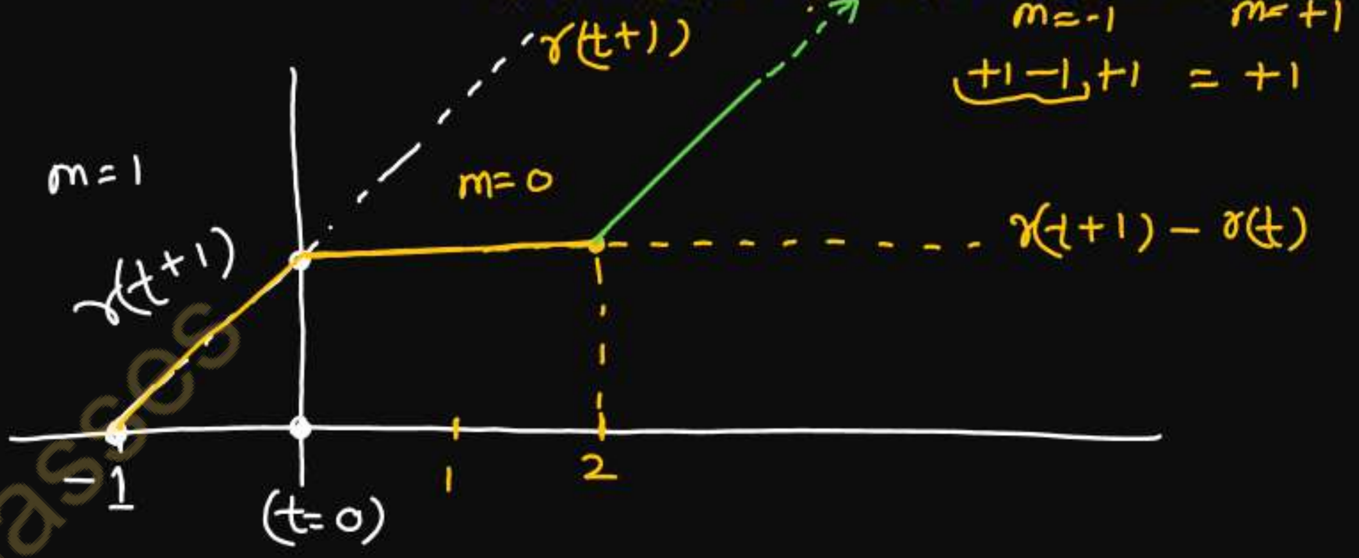


$m=+1$

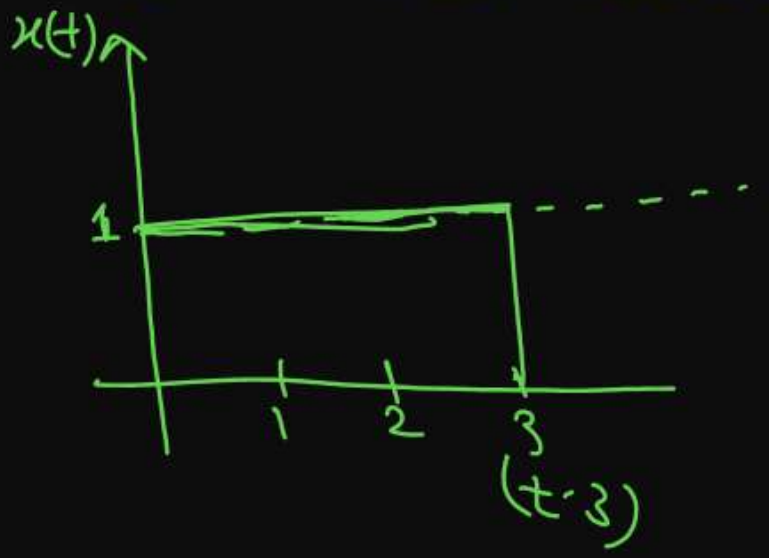
(ii) $x(t)= r(t+1) - r(t) + r(t-2)$

$m=-1$ $\quad m=+1$

$\underbrace{+1-1}+1 = +1$

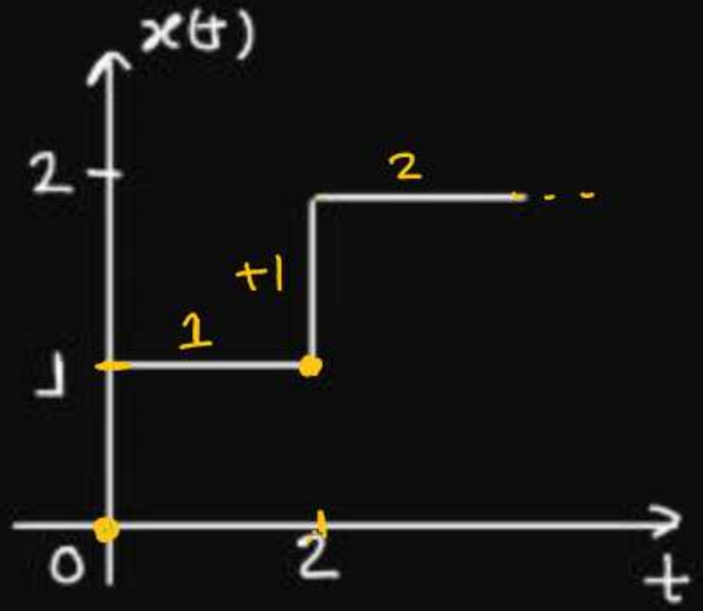


(iii) $x(t) = u(t) - u(t-3)$



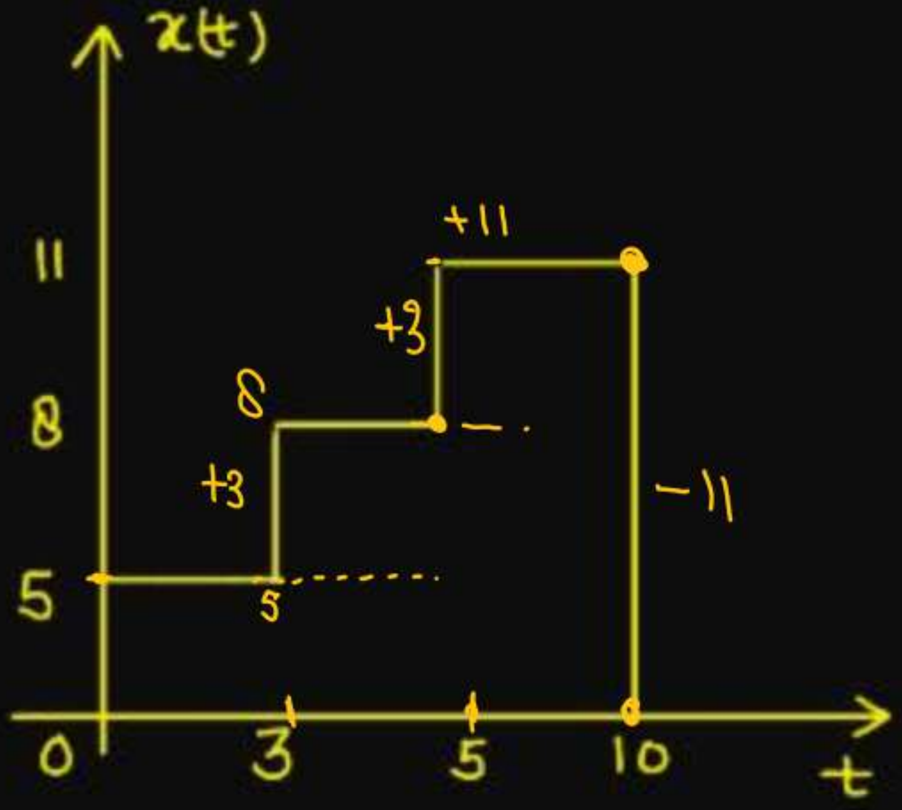
Operation on Elementary Signals (Properties)

Ex2



$x(t) = u(t) + 1U(t-2)$

Ex.

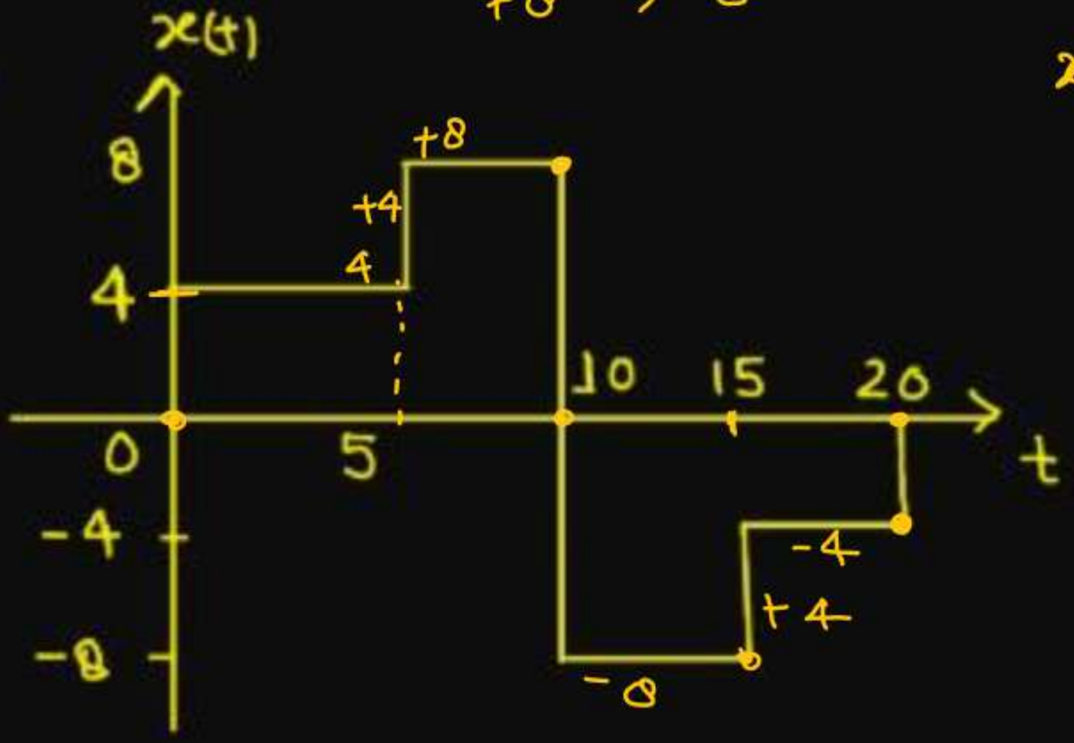


$x(t) = 5U(t) + 3U(t-3) + 3U(t-5) - 11U(t-10)$

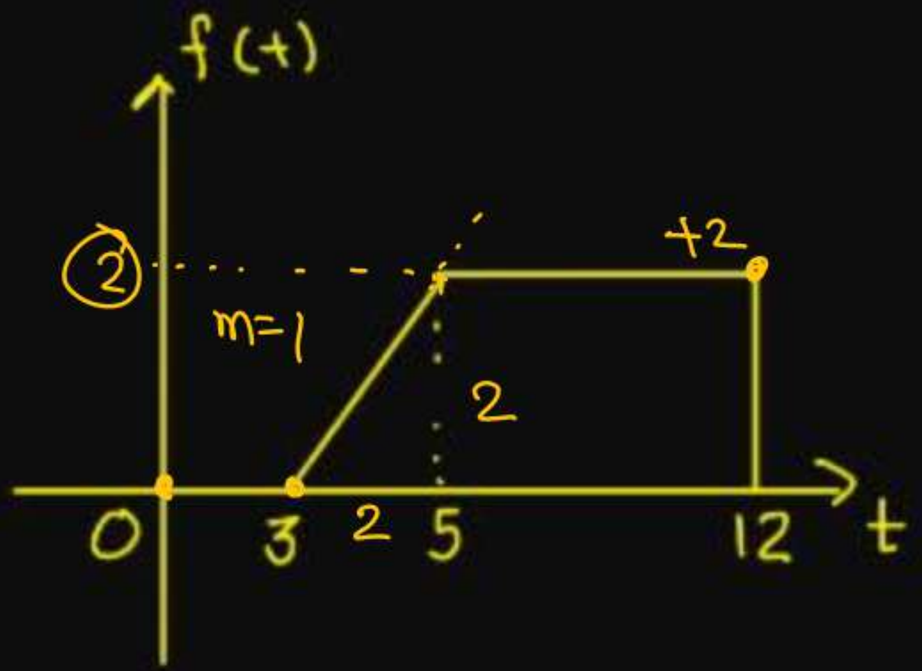
Operation on Elementary Signals (Properties)

Ex3

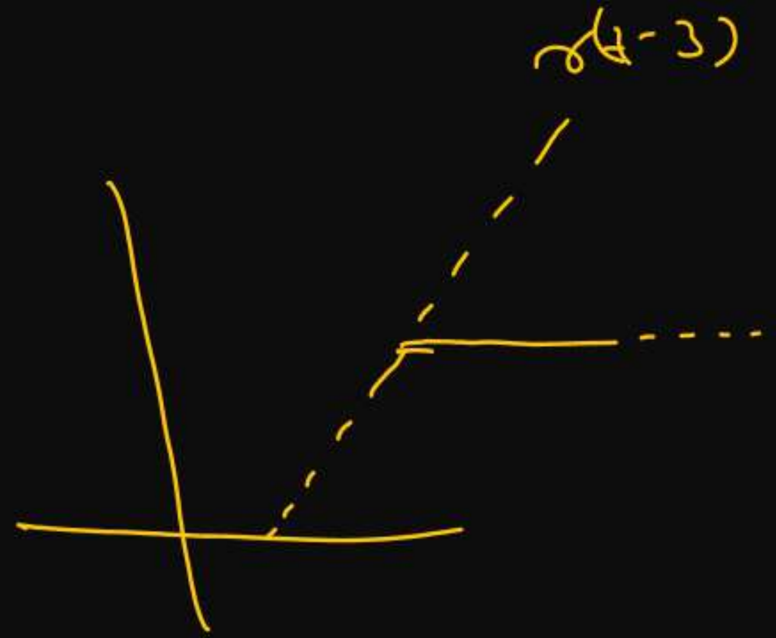
$+8 \rightarrow -8$



$$x(t) = 4u(t) + 4u(t-5) - 16u(t-10) + 4u(t-15) + 4u(t-20)$$

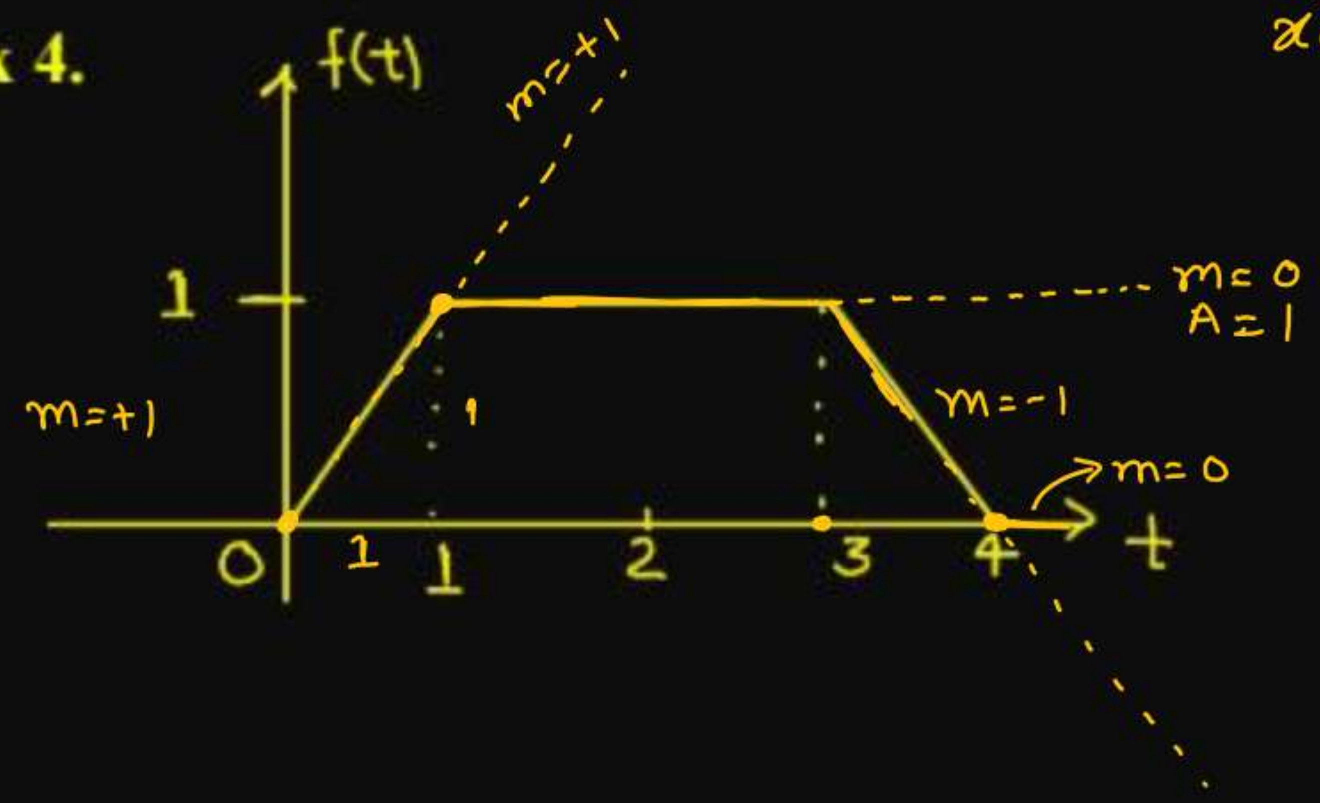


$$x(t) = r(t-3) - r(t-5) - 2u(t-12)$$

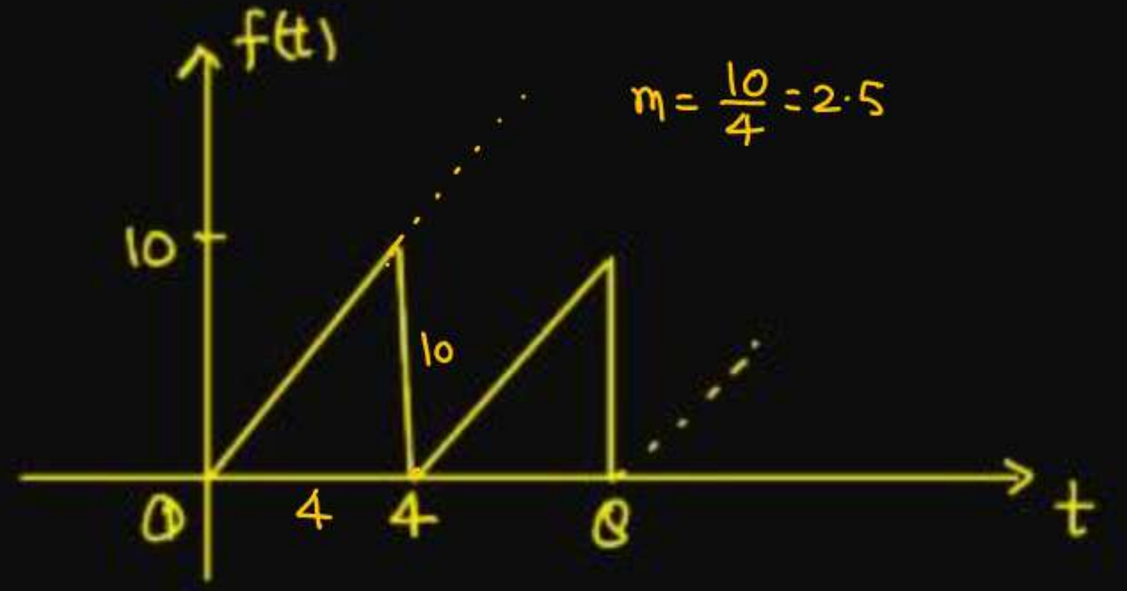


Operation on Elementary Signals (Properties)

Ex 4.

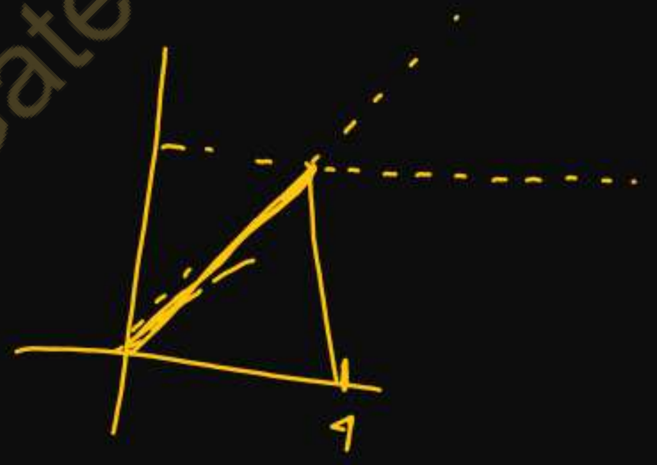

f(t)
m=+1
1
m=+1
1
m=0
A=1
m=-1
m=0
0
1
1
2
3
4
t

$x(t) = r(t) - r(t-1) - r(t-3) + r(t-4)$


f(t)
$m = \frac{10}{4} = 2.5$
10
10
0
4
4
8
t

$\overbrace{m=+2.5 \qquad m=-2.5}^{0 \to \text{step}}$

$x(t) = 2.5\,r(t) - 2.5\,r(t-4) - 10\,u(t-4) + \ldots$



AKTU

B.Tech Sem-III

GW
GATEWAY CLASSES

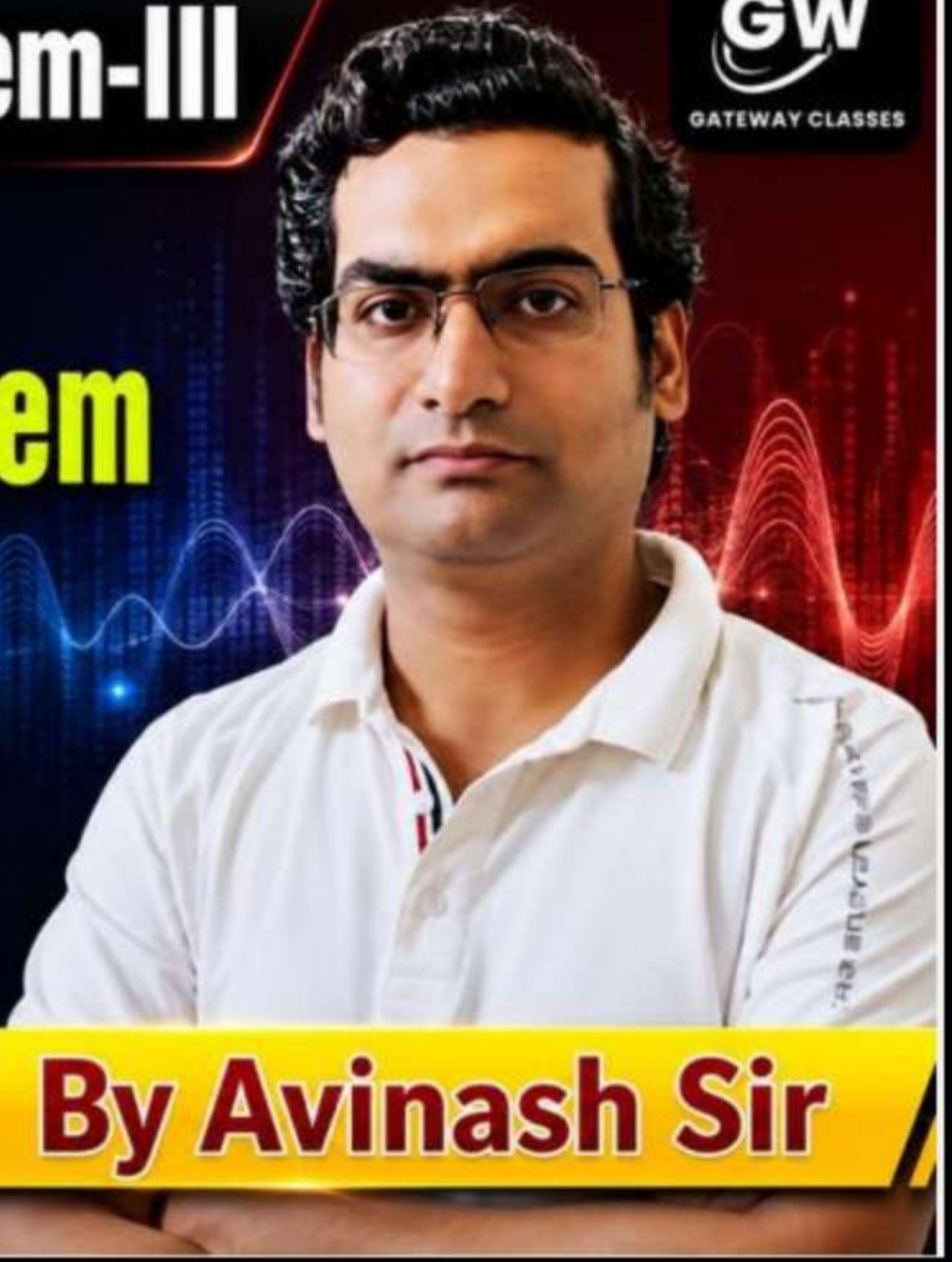
EE and EEE Branch

BSS: Basics Signal System

Unit-1 : Lec-2

Topics to be covered

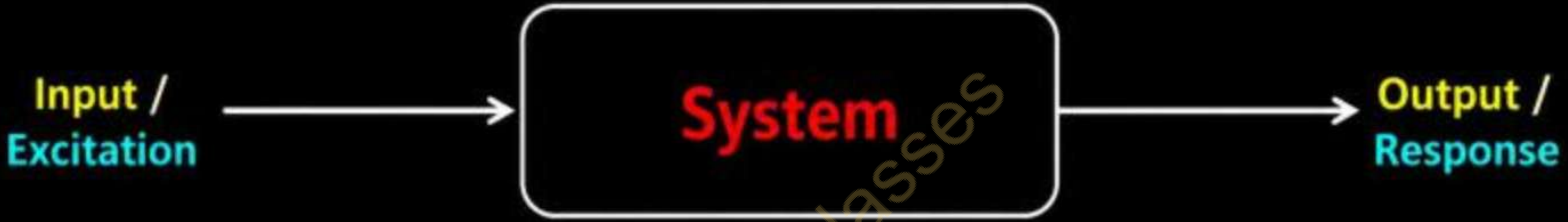
- Classification of System
- Properties of System
- AKTU PYQs



By Avinash Sir

What is a System ?

- A collection of physical, biological or abstract components which together perform an intended objective.
- A system gives an output (also called response) for an input (also called excitation).

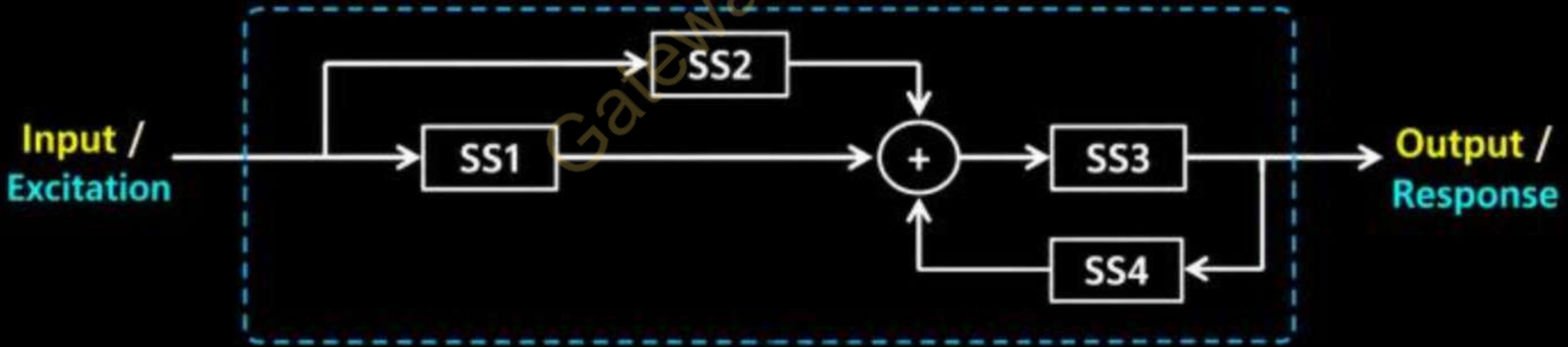


Input / Excitation

System

Output / Response

- System can be a collection of multiple of sub systems



Input / Excitation

SS1

SS2

+

SS3

SS4

Output / Response

System (with 4 sub-systems)

Classification of System

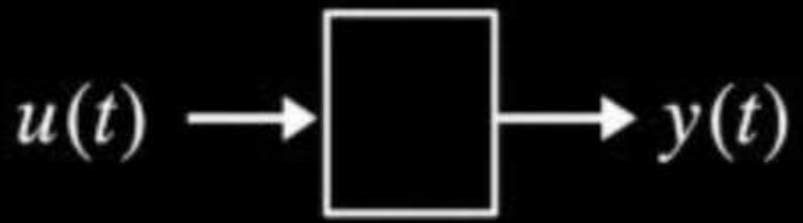
Classification of System

- Continuous time and Discrete time system
- Linear and Non-Linear system
- Static and Dynamic system
- Causal and Non-Causal system
- Time invariant and Time variant system
- Stable and Unstable system

Continuous time and Discrete time system

- **Continuous time system:** Continuous time system operates on a continuous time signal (input or excitation) and produces another continuous time signal (output or response).
- the input signal u(t) is transformed by the system into signal y(t),this transformation can be expressed as,

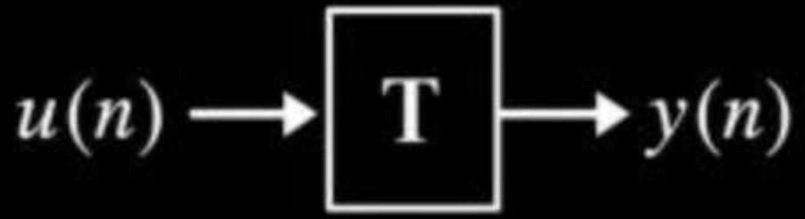
Response $y(t) = F[u(t)]$



Discrete time system: system operates on a discrete time signal (input or excitation) and produces another discrete time signal (output or response).

The signal u(n) is transformed by the system into signal $y(n)$, this transformation can be expressed as,

Response $y(n) = F[u(n)]$



Linear and Non- Linear System

- **Linear System:-** A system is said to be linear if it satisfies the properties of homogeneity and superposition.

(a) **Superposition Property:-** According to this property if we have two inputs such that,

For $x_1(t)$ output is $y_1(t)$ & For $x_2(t)$ output is $y_2(t)$

Then, $\boxed{x_1(t) + x_2(t) = y1(t) + y_2(t)}$

(b) **Homogeneity:-** According to this principle for an arbitrary input x(t) and an arbitrary scalar a, if

$$x(t) \longrightarrow y(t)$$

Then, $\boxed{ax(t) \longrightarrow ay(t)}$

Examples on Linear system

- Determine whether the given system is linear or not
- **Ex. 1 $y(t)= [x(t)^2]$**

Test for Additivity

Consider two input signals:

$$x_1(t) \to y_1(t)$$

and

$$x_2(t) \to y_2(t)$$

For the given system,

$$y_1(t) = [x_1(t)]^2$$
$$y_2(t) = [x_2(t)]^2$$

Adding the individual outputs,

$$y_1(t) + y_2(t) = [x_1(t)]^2 + [x_2(t)]^2$$

Apply Combined Input

Let the input be

$$x(t) = x_1(t) + x_2(t)$$

Then the corresponding output is

$$y(t) = [x_1(t) + x_2(t)]^2$$

Expanding,

$$y(t) = [x_1(t)]^2 + [x_2(t)]^2 + 2x_1(t)x_2(t)$$

Compare Outputs

From additivity,

$$y_1(t) + y_2(t) = [x_1(t)]^2 + [x_2(t)]^2$$

But,

$$y(t) = [x_1(t)]^2 + [x_2(t)]^2 + 2x_1(t)x_2(t)$$

Therefore,

$$y(t) \neq y_1(t) + y_2(t)$$

because of the extra term

$$2x_1(t)x_2(t)$$

Since the additivity property is not satisfied,

$$\boxed{y(t) = [x(t)]^2}$$

does **not** satisfy the superposition principle.

Hence,

$$\boxed{\text{The } \mathit{system\ is\ Non{-}Linear}}$$

Examples on Linear system

- Determine whether the given system is linear or not
- **Ex. 1 $y(t) = e^t x(t)$**

Test for Additivity

Consider two input signals:

$$x_1(t) \rightarrow y_1(t), \quad x_2(t) \rightarrow y_2(t)$$

For the given system,

$$y_1(t) = e^t x_1(t)$$

$$y_2(t) = e^t x_2(t)$$

Adding the outputs,

$$y_1(t) + y_2(t) = e^t x_1(t) + e^t x_2(t)$$

$$= e^t[x_1(t) + x_2(t)]$$

Apply Combined Input

$$x(t) = x_1(t) + x_2(t)$$

Then the output becomes

$$y(t) = e^t[x_1(t) + x_2(t)]$$

$$= e^t x_1(t) + e^t x_2(t)$$

$$= y_1(t) + y_2(t)$$

Thus, $\boxed{y(t) = y_1(t) + y_2(t)}$

Hence, the **additivity property is satisfied**.

Test for Homogeneity (Scaling): Let the input be scaled by a constant a: $x(t) \rightarrow a\,x(t)$

The output is $y(t) = e^t[a\,x(t)]$

$$= a\,e^t x(t) = a\,y(t)$$

Thus, $\boxed{T\{a\,x(t)\} = a\,T\{x(t)\}}$

Hence, the **homogeneity property is satisfied**.

Determine whether the given system y(t)= t + 5 is linear or not

Test for Additivity: Consider two input signals:

$x_1(t) \rightarrow y_1(t)$, $x_2(t) \rightarrow y_2(t)$

For the given system,

$$y_1(t) = x_1(t) + 5$$

$$y_2(t) = x_2(t) + 5$$

Adding the individual outputs,

$$y_1(t) + y_2(t) = x_1(t) + x_2(t) + 10$$

Apply Combined Input

Let, $x(t) = x_1(t) + x_2(t)$

Then the output becomes

$$y(t) = [x_1(t) + x_2(t)] + 5$$

$$y(t) = x_1(t) + x_2(t) + 5$$

Compare Results: From individual outputs:

$$y_1(t) + y_2(t) = x_1(t) + x_2(t) + 10$$

From combined input: $y(t) = x_1(t) + x_2(t) + 5$

Clearly, $y(t) \neq y_1(t) + y_2(t)$

Therefore, the **additivity property is violated**.

Test for Homogeneity: Let the input be scaled by a constant a: $x(t) \rightarrow a\,x(t)$

Output: $T[a\,x(t)] = a\,x(t) + 5$

But $a\,T[x(t)] = a[x(t) + 5] = a\,x(t) + 5a$

Since $a\,x(t) + 5 \neq a\,x(t) + 5a$

So, the homogeneity property is also violated.

So, The system does not satisfy the superposition principle Hence, $\boxed{\text{The } system\ is\ Non\text{-}Linear}$

- Determine whether the given system **y(t)= t sint** is linear or not

Test for Additivity : Consider two input signals:

$x_1(t) \rightarrow y_1(t)$, $x_2(t) \rightarrow y_2(t)$

For the given system,

$$y_1(t) = \sin(x_1(t+2))$$

$$y_2(t) = \sin(x_2(t+2))$$

Adding the outputs,

$$y_1(t) + y_2(t) = \sin(x_1(t+2)) + \sin(x_2(t+2))$$

Apply Combined Input

Let, $x(t) = x_1(t) + x_2(t)$

Then the output is $y(t) = \sin(x_1(t+2) + x_2(t+2))$

Compare Results, For additivity to hold,

$$\sin(x_1(t+2) + x_2(t+2))$$

$$= \sin(x_1(t+2)) + \sin(x_2(t+2))$$

But in general,

$$\sin(a+b) \neq \sin a + \sin b$$

Therefore, $y(t) \neq y_1(t) + y_2(t)$

Hence, the **additivity property is violated**.

Test for Homogeneity

Let the input be scaled by a constant k:

$$x(t) \rightarrow kx(t)$$

Output:

$$T[kx(t)] = \sin(kx(t+2))$$

For homogeneity to hold,

$$T[kx(t)] = k\,T[x(t)]$$

i.e., $\sin(kx(t+2)) = k\sin(x(t+2))$

This is not true in general.

Therefore, homogeneity is also violated.

Static Vs Dynamic Systems

- **Static systems:** At any time, output of the system depends only on present input.
- Memory Less System
- Response $y(t) = F[u(t)]$
- E.g. Resistor

Dynamic system: At any time, output of the system depends on present as well as past inputs.

- Presence of Memory can be observed.
- Response $y(t) = 2x(t) + x(-t)$
- E.g. Inductor

Causal and Non Causal System

Causal Systems

- Output is only dependent on inputs already received (present or past)
- Non anticipatory system
- $Y(t)=f\ x(t), x(t-1), \ldots\ldots$
- E.g.

  Thermostat based AC

Non- Causal Systems

- Output is dependent on future inputs as well
- System anticipates future inputs based on past
- $Y(t)=f\ x(t), x(t+1) \ldots\ldots$
- E.g.

  Weather Forecasting system

  Missile guidance System

Time Invariant Vs Time Variant Systems

Time Invariant Systems

- Output of the system is independent of the time at which the input is applied
- Y(t)= f(u(t))= y(t+δ)= fu(t+ δ)
- E.g. An ideal resistor

$$I(t) = \frac{V(t)}{R} \Rightarrow I(t+\delta)\,\frac{V(t+\delta)}{R}$$

Time Variant Systems

- Output of the system is dependent on the time at which the input is applied
- Y(t)= f(u(t))≠ y(t+δ) ≠ fu(t+ δ)
- E.g. Aircraft : Mass (M) of aircraft changes as fuel is consumed

- Determine whether the given system **$y(t)= [x(t)^2]$** is Time Variant or not

Step 1: Apply a Delay to the Input

Let the input be delayed by t_0:

$$x(t) \rightarrow x(t - t_0)$$

The output corresponding to this delayed input is

$$y_1(t) = [x(t - t_0)]^2$$

Step 2: Delay the Original Output

The original output is $y(t) = [x(t)]^2$

Replacing t by $(t - t_0)$,

$$y(t - t_0) = [x(t - t_0)]^2$$

Step 3: Compare the Results

From Step 1, $y_1(t) = [x(t - t_0)]^2$

From Step 2, $y(t - t_0) = [x(t - t_0)]^2$

Therefore, $\boxed{y_1(t) = y(t - t_0)}$

Since delaying the input by t_0 causes an identical delay in the output,

$$T\{x(t - t_0)\} = y(t - t_0)$$

the system satisfies the time-invariance condition.

Hence,

$$\boxed{y(t) = [x(t)]^2}$$

is a

$$\boxed{\text{Time−Invariant } System}$$

- Determine whether the given system **$y(t)= t\,x(t) + \frac{dx(t)}{dt}$** is Time Variant or not

Step 1: Delay the Input Signal

Let the input be delayed by t_0:

$$x(t) \rightarrow x(t-t_0)$$

The output corresponding to the delayed input is

$$y_1(t) = t\,x(t-t_0) + \frac{d}{dt}[x(t-t_0)]$$

Step 2: Delay the Original Output

The original output is

$$y(t) = t\,x(t) + \frac{dx(t)}{dt}$$

Replacing $t$by $(t-t_0)$,

$$y(t-t_0) = (t-t_0)x(t-t_0) + \frac{d}{dt}[x(t-t_0)]$$

Step 3: Compare the Results

From Step 1: $y_1(t) = t\,x(t-t_0) + \frac{d}{dt}[x(t-t_0)]$

From Step 2: $y(t-t_0) = (t-t_0)x(t-t_0) + \frac{d}{dt}[x(t-t_0)]$

Clearly,

$$t\,x(t-t_0) \neq (t-t_0)x(t-t_0)$$

Therefore, $\boxed{y_1(t) \neq y(t-t_0)}$

Since a delay in the input does **not** produce an identical delay in the output, $T\{x(t-t_0)\} \neq y(t-t_0)$

the system fails the time-invariance test.

Hence, Given System is

$$\boxed{\text{Time–Variant } System}$$

- Determine whether the given system **$y(t)= e^t x(t)$** is Time Variant or not

Step 1: Delay the Input Signal

Let the input be delayed by t_0:

$$x(t) \rightarrow x(t - t_0)$$

The output corresponding to the delayed input is

$$y_1(t) = e^t\, x(t - t_0)$$

Step 2: Delay the Original Output

The original output is

$$y(t) = e^t\, x(t)$$

Replacing $t$by $(t - t_0)$,

$$y(t - t_0) = e^{(t-t_0)}\, x(t - t_0)$$

$$= e^t e^{-t_0}\, x(t - t_0)$$

Step 3: Compare the Results

From Step 1: $y_1(t) = e^t\, x(t - t_0)$

From Step 2: $y(t - t_0) = e^{t-t_0}\, x(t - t_0)$

Since

$$e^t \neq e^{t-t_0}$$

for $t_0 \neq 0$,

$$\boxed{y_1(t) \neq y(t - t_0)}$$

Since delaying the input does not simply delay the output,

$$T\{x(t - t_0)\} \neq y(t - t_0)$$

the system fails the time-invariance test.

Therefore, $\boxed{y(t) = e^t\, x(t)}$

is a $\boxed{\text{Time−Variant } System}$

- **Determine whether the system y(t) = t·x(t) + x(−t) is linear and time invariant.**

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1. Test for Linearity: A system is linear if it satisfies the **superposition principle**.

Step 1: Consider Two Inputs

Let $x_1(t) \to y_1(t)$, $x_2(t) \to y_2(t)$

Then

$$y_1(t) = t\, x_1(t) + x_1(-t)$$

$$y_2(t) = t\, x_2(t) + x_2(-t)$$

Adding the outputs, $y_1(t) + y_2(t)$

$$= t[x_1(t) + x_2(t)] + [x_1(-t) + x_2(-t)]$$

Step 2: Apply Combined Input

Let $x(t) = x_1(t) + x_2(t)$

Then $y(t) = t[x_1(t) + x_2(t)] + [x_1(-t) + x_2(-t)]$

$$y(t) = y_1(t) + y_2(t)$$

Thus, **additivity is satisfied**.

Step 3: Check Homogeneity: Let the input be scaled by a constant a: $x(t) \to a\, x(t)$

Output: $T\{a\, x(t)\} = t[a\, x(t)] + a\, x(-t)$

$$= a[t\, x(t) + x(-t)] = a\, T\{x(t)\}$$

Thus, **homogeneity is satisfied** . The *system is Linear*

- **Determine whether the system y(t) = t·x(t) + x(−t) is linear and time invariant.**

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Test for Time Invariance

A system is time-invariant if

$$T\{x(t-t_0)\} = y(t-t_0)$$

Step 1: Delay the Input

Let, $x(t) \rightarrow x(t-t_0)$

The corresponding output is

$$y_1(t) = t\,x(t-t_0) + x(-(t)+t_0)$$

$$y_1(t) = t\,x(t-t_0) + x(t_0-t)$$

Step 2: Delay the Original Output

Original output: $y(t) = t\,x(t) + x(-t)$

Replace $t$by $(t-t_0)$:

$$y(t-t_0) = (t-t_0)x(t-t_0) + x\big(-(t-t_0)\big)$$

$$y(t-t_0) = (t-t_0)x(t-t_0) + x(t_0-t)$$

Step 3: Compare

From Step 1: $y_1(t) = t\,x(t-t_0) + x(t_0-t)$

From Step 2: $y(t-t_0) = (t-t_0)x(t-t_0) + x(t_0-t)$

Clearly, $t\,x(t-t_0) \neq (t-t_0)x(t-t_0)$

Therefore,

$$\boxed{y_1(t) \neq y(t-t_0)}$$

So The *system is Time Variant*

Stable and unstable system

A system is said to be stable if and only if it satisfies the BIBO stability criterion.

BIBO stable condition:

- Every bounded input yields bounded output.

  i. e. , if $0 < u(t) < \infty$ *then* $0 < y(t) < \infty$ should be satisfied for the system to be stable.

- Impulse response should be absolutely integrable

$$\text{i.e.,}\ 0 < \int_{-\infty}^{\infty} |\, h(\tau)\, |\, d\tau < \infty$$

If the BIBO stable condition is not satisfied, then the system is said to be ***unstable system***

Linear Time Invariant System

- A system for which the principle of superposition and the principle of homogeneity are valid.
- Input/output characteristics do not with time.

Properties of LTI System

- Commutative
- Distributive
- Associative
- Invertibility
- Static and Dynamic System
- Causality
- Stability

Properties of LTI System

Commutative :-

Distributive :-

Associative:-

Properties of LTI System

Invertibility:- A system is said to be invertible if there exists an inverse system which when connected in series with the original system produces an output identical to the input.

Static and Dynamic System:- These are memory less systems. Output of system depends upon input of system at present time only.

Causality:- A causal system depends only on the present and past values of the input to the system. Impulse response h(t) is zero for $t < 0$

Stability:- A system is stable if it has bounded input and bounded output.

AKTU **B.Tech Sem-III**

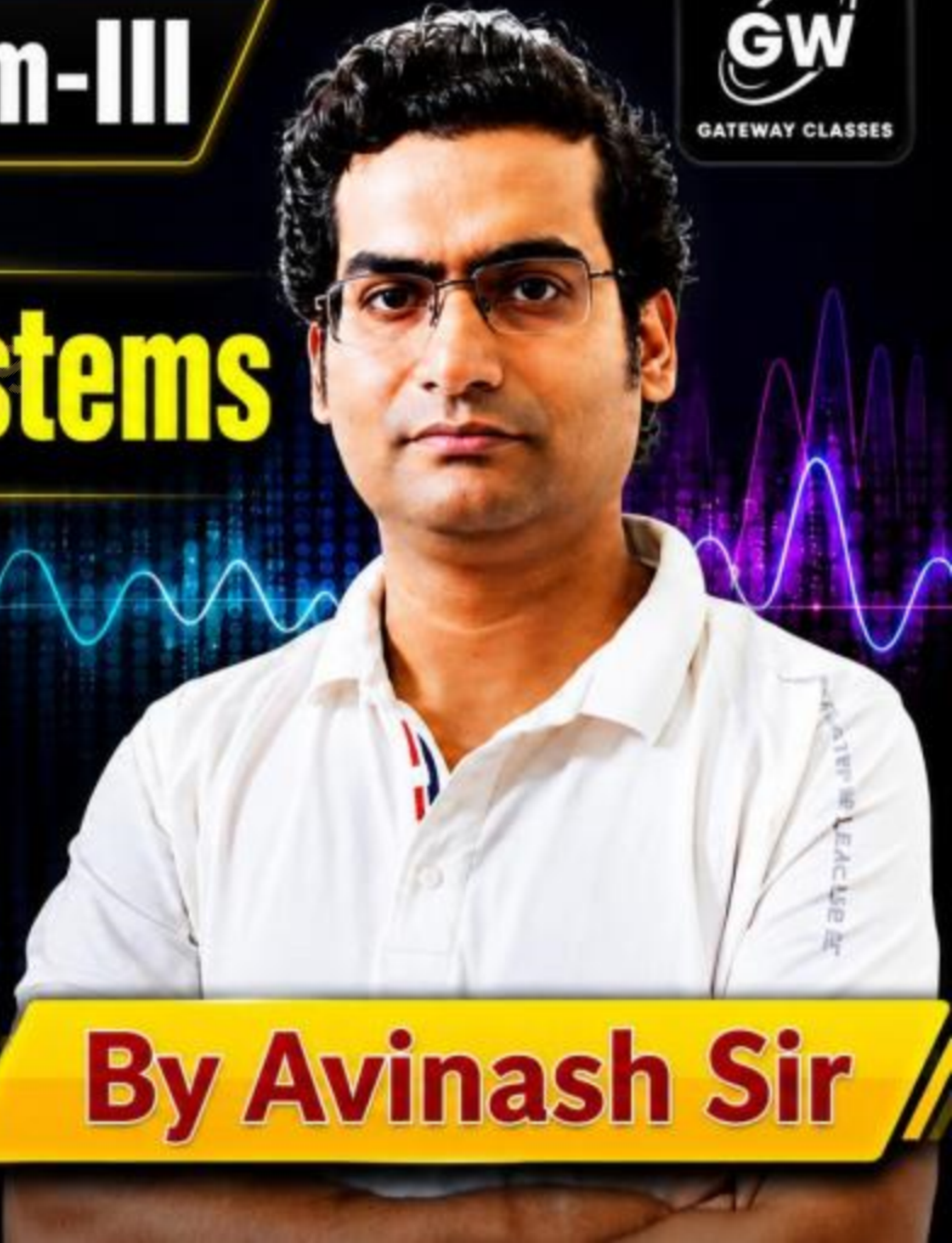
GATEWAY CLASSES

BSS: Basic Signals & Systems

Unit-1 : Lec-3

Topics to be covered

- Analogy between Electrical and Mechanical System
- PyQ



By Avinash Sir

Mathematical Modelling of Electro-Mechanical System

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Translational System: It has 3 types of Resisting forces due to elements.

1. Inertial Force: Due to inertial mass,

$$F_M(t) = M.a = M.\frac{du}{dt} = M.\frac{d^2x}{dt^2}$$

M= Mass, x=Linear displacement, u= Linear velocity, a= Acceleration

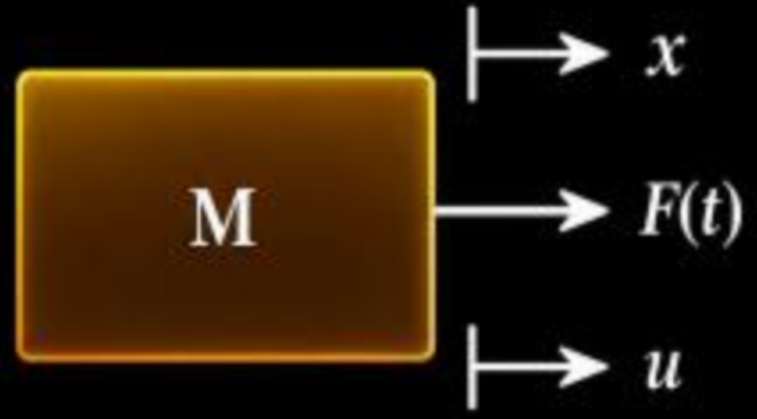
2. Damping Force [Viscous Damping]: Due to viscous damping, it is proportional to velocity & is given by, $F_D \propto u$ (velocity)

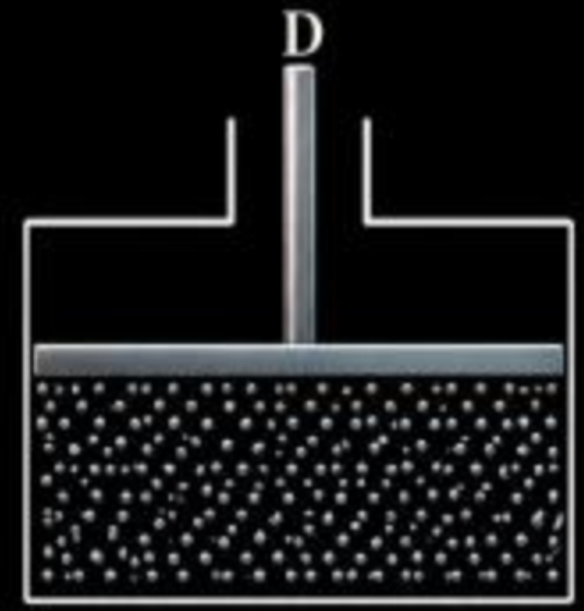
$$F_D(t) = B.u = B.\frac{dx}{dt}$$

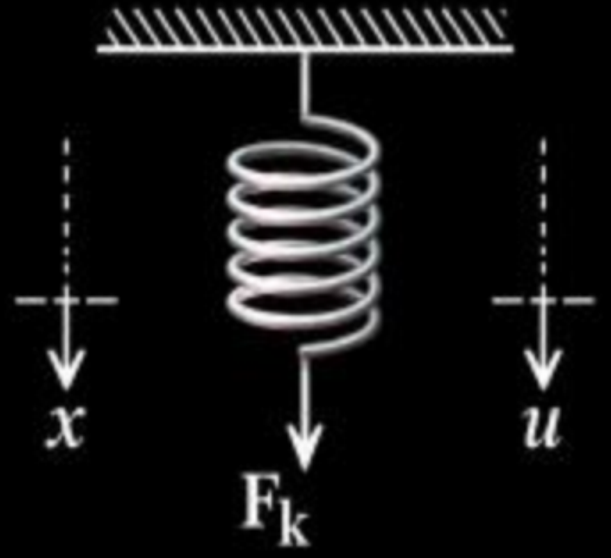
Damping force is denoted by either D or B or F

3. Spring Force: Spring force is proportional to displacement.

$$F_K(t) = K.x$$

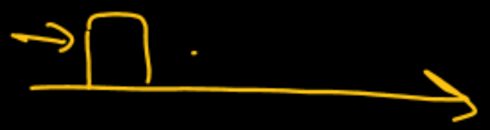




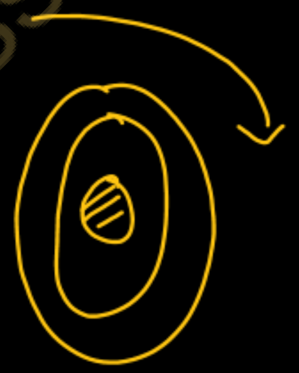


Mechanical system

Translational system



Rotational System



Mathematical Modelling of Electro-Mechanical System

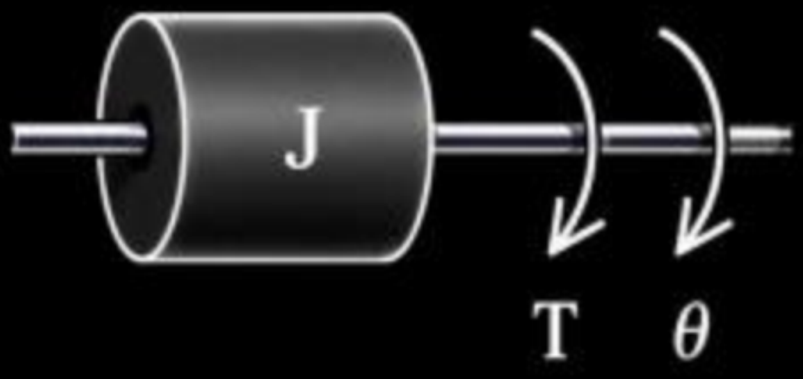
Rotational System: *It also has 3 types of forces due to elements.*

$x \rightarrow \theta$

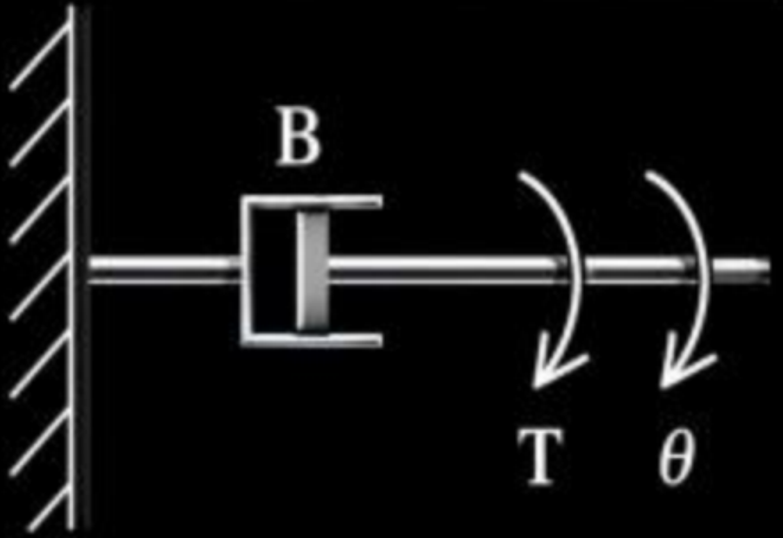
1. Inertial Torque:

$$\underline{T = J\frac{d^2\theta}{dt^2}} = J\frac{d\omega}{dt}$$

Where T= Torque due to inertia, J= Inertia constant, θ= angular displacement , ω = *angular speed*



2. Damping Torque: If a torque is applied on damper B, then it is opposed by an opposing torque due to the rotational friction of the dashpot. This opposing torque is proportional to the angular velocity of the body.



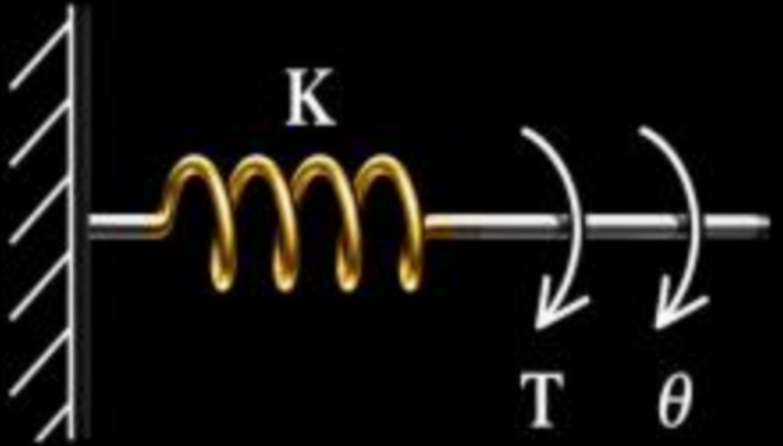
$$\underline{T_D = B\frac{d\theta}{dt}} = B.\omega$$

Where , B = Damping constant

3. Spring Torque: In rotational mechanical system, torsional spring stores **potential energy**.

$$\underline{T = K\theta} = K.\int\omega\, dt$$

Where , K = spring constant



D'Alembert's Principle: The static equilibrium of a dynamic system subjected to an external driving force obeys the following principle, "For any body, the algebraic sum of externally applied forces resisting motion in any given direction is zero".

Translational Mechanical System

Applied force: $F = F_M + F_B + F_K$

Hence,

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

Standard Equation

$$\boxed{F(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Rotational Mechanical System

Applied Torque $T = T_J + T_B + T_K$

Therefore,

$$\boxed{T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

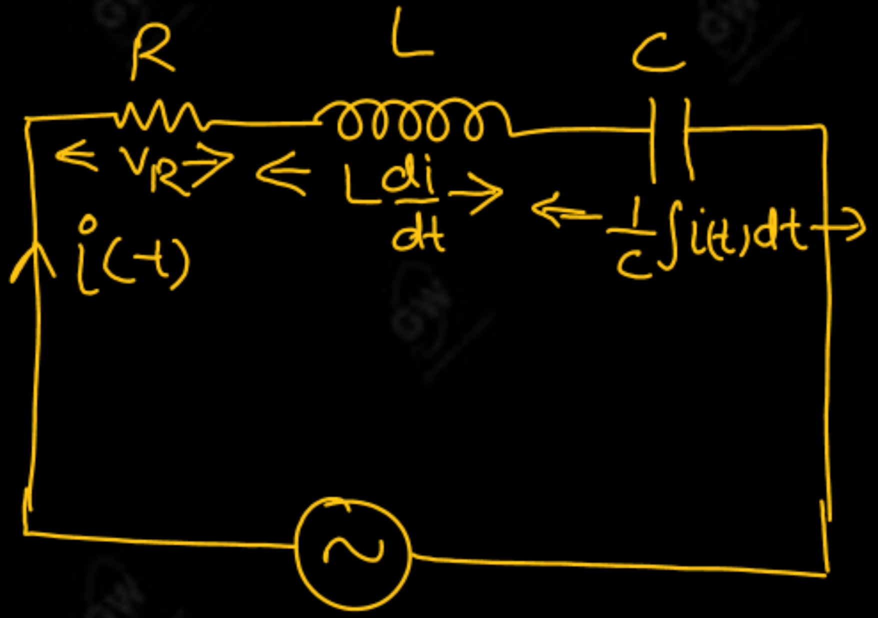
Where:

- J= Moment of inertia
- B= Viscous friction coefficient
- K= Torsional spring constant

Modelling of Electric circuit (Analogous System)

Series RLC System:-

Series RLC Analogy (Force-Voltage Analogy)


R
L
C
$\leftarrow V_R \rightarrow$
$\leftarrow L\frac{di}{dt} \rightarrow$
$\leftarrow \frac{1}{C}\int i(t)\,dt \rightarrow$
$i(t)$
~
V

Apply KVL:

$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\ dt$$

F, T, V

F-V Analogy ✓

T-V Analogy

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\cdot\theta$$

Since

$$i = \frac{dq}{dt}$$

$$V = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{q}{C}$$

or

$$\boxed{V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q} \quad\text{...............(3)}$$

F — V — T

Mechanical	**Electrical**	Mechanical
Force F	**Voltage V**	Torque T
Mass M	**Inductance L**	Inertia J
Damper B	**Resistance R**	Damper B
Spring K	**$1/C$**	Spring K
Displacement x	**Charge q**	Displacement θ

Modelling of Electric circuit (Analogous System)

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Parallel RLC System:-

Apply KCL:

$$I = I_R + I_L + I_C$$

$$I = \frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt}$$

Since

$$V = \frac{d\phi}{dt}$$

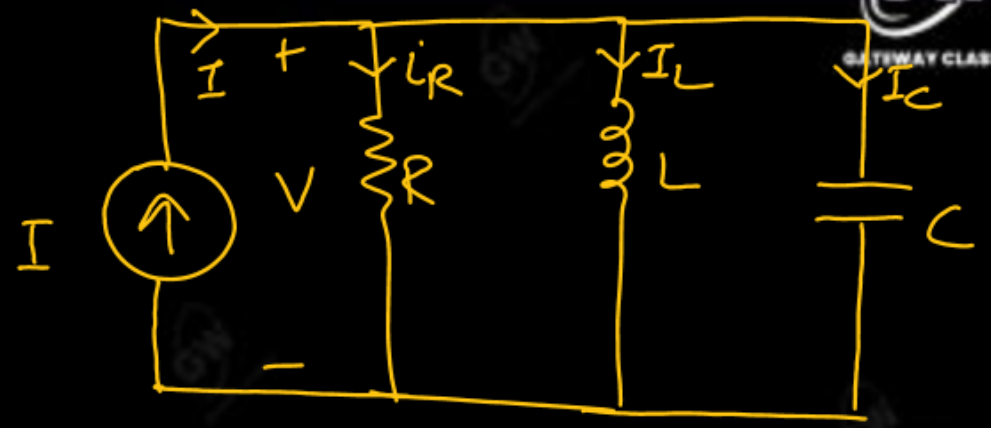
$$I = \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi + C\frac{d^2\phi}{dt^2}$$

Thus

$$\boxed{I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi}$$

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta$$



f-I Analogy (T-I) Analogy

F I T

Mechanical	Electrical	Mechanical
Force F	Current I	Torque T
Mass M	Capacitance C	Inertia J
Damper B	Conductance $1/R$	Damper B
Spring K	$1/L$	Spring K
Displacement x	Flux linkage ϕ	Displacement θ

Analogy between Electrical and Mechanical System

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T–V An.

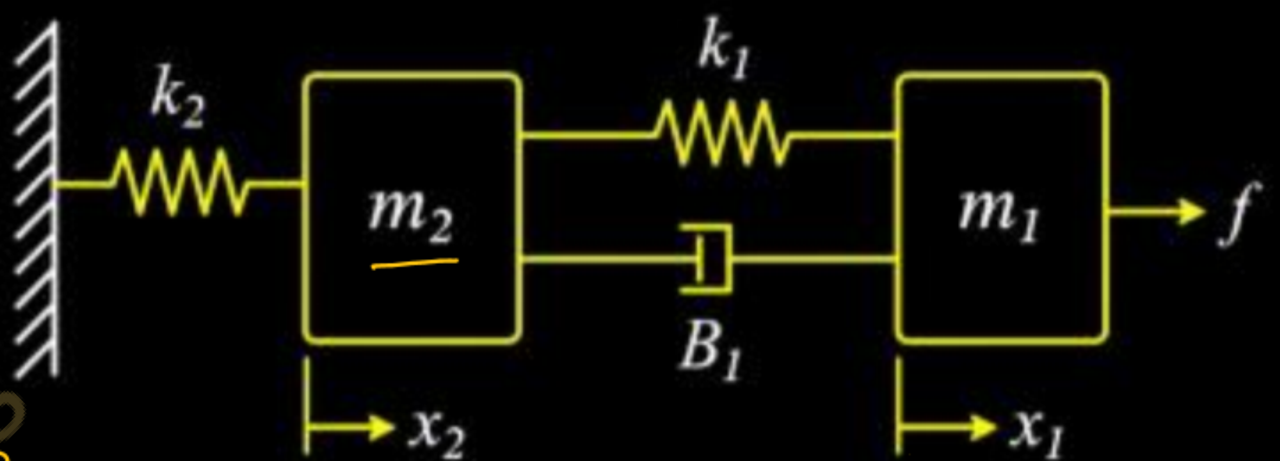
F V

F I

T–I

S.No.	Mechanical System (Translational)	Electrical System (Series)	Electrical System (Parallel)	Mechanical System (Rotational)
1	Force (F)	Voltage (V)	Current (I)	Torque (T)
2	Mass (M)	Inductance (L)	Capacitance (C)	Moment of inertia Constant (J)
3	Damping Coefficient (B)	Resistance (R)	Conductance G=1/R	Rotational friction coefficient (B)
4	Spring Constant Stiffness (K)	Elastance (1/C)	Susceptance (1/L)	Torsional Spring Constant (K)
5	Displacement (x)	Charge (q)	Magnetic Flux (Φ)	Angular Displacement (Θ)

Ex. 1 Write down the differential equation governing the mechanical translation system and Draw analogous electrical System using F-V Analogy.


k_2
k_1
m_2
m_1
f
B_1
x_2
x_1

Sol^n from 1st F.B.D.

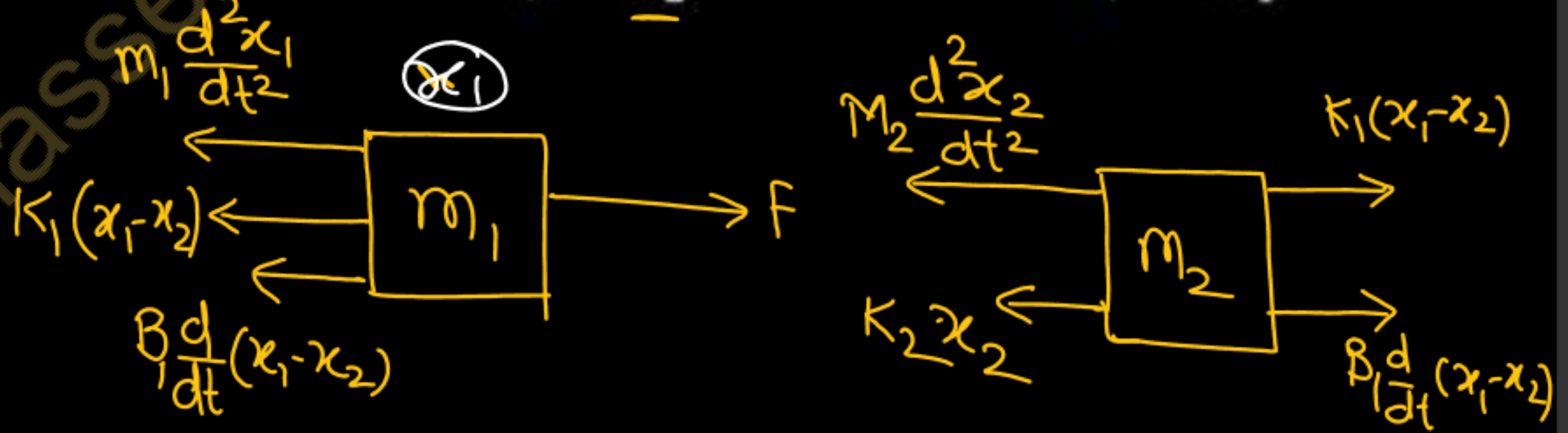
$$F = M_1\frac{d^2x_1}{dt^2} + B_1\frac{d(x_1-x_2)}{dt} + K_1(x_1-x_2) \quad \text{—①}$$

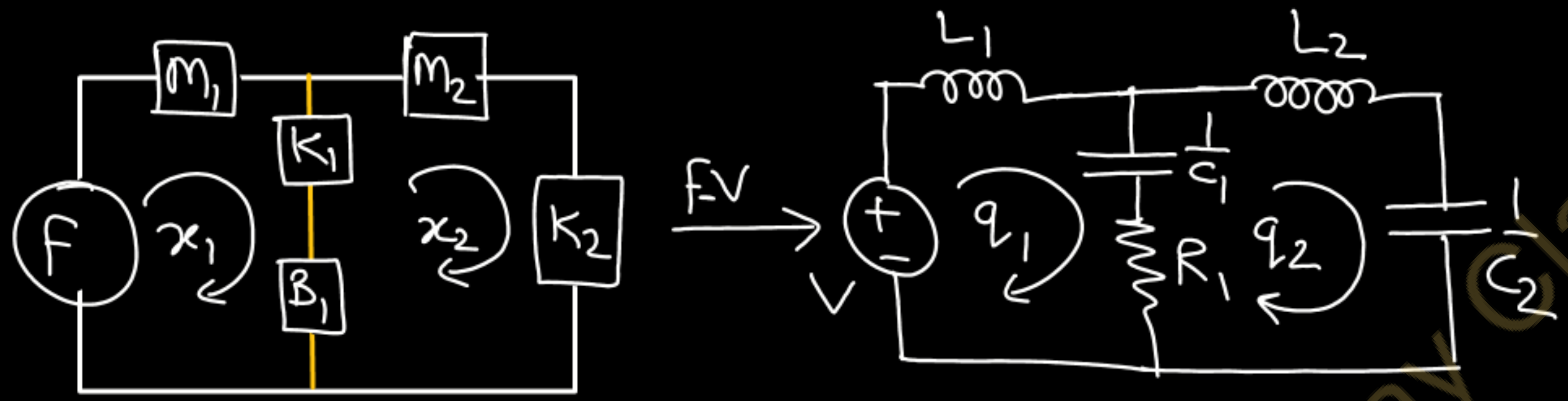
{No. of Mass Disp. = No. of Loop} f-V

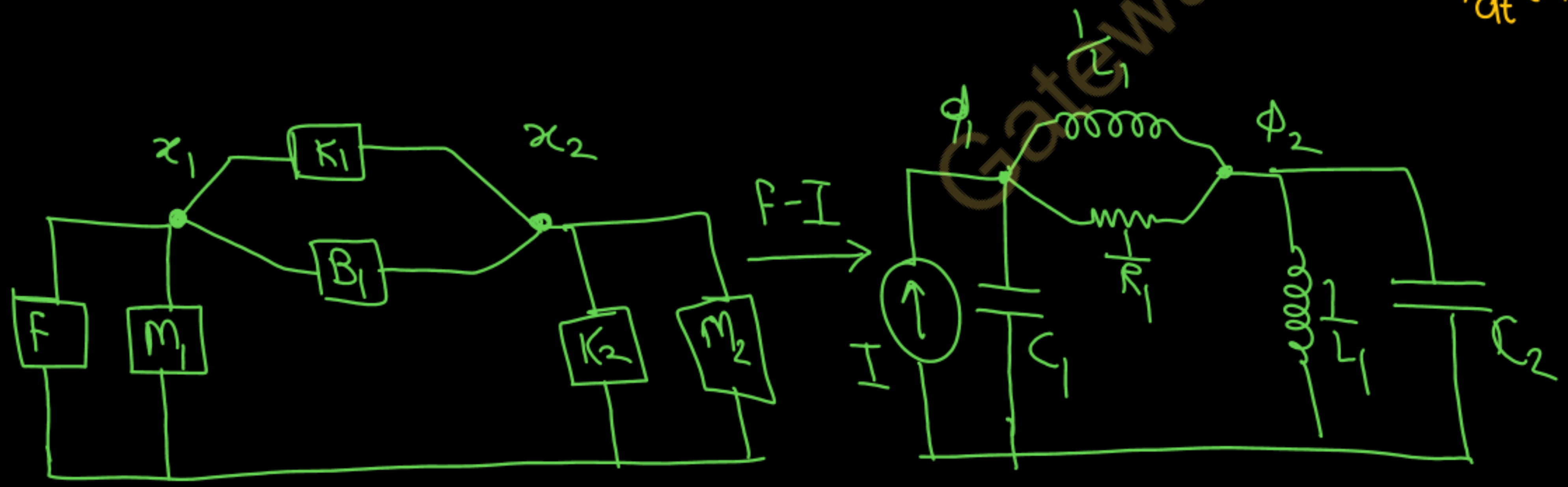
{No. disp. = No. of Node} f-I

From FBD-2

$$K_1(x_1-x_2) + B_1\frac{d}{dt}(x_1-x_2) = M_2\frac{d^2x_2}{dt^2} + K_2x_2 \quad \text{—②}$$


$M_1\frac{d^2x_1}{dt^2}$
x_1
$K_1(x_1-x_2)$
m_1
F
$B_1\frac{d}{dt}(x_1-x_2)$
$M_2\frac{d^2x_2}{dt^2}$
$K_1(x_1-x_2)$
m_2
K_2x_2
$B_1\frac{d}{dt}(x_1-x_2)$


M_1
M_2
K_1
B_1
F
x_1
x_2
K_2
F-V
L_1
L_2
$\frac{1}{C_1}$
R_1
q_1
q_2
$\frac{1}{C_2}$
V


x_1
x_2
K_1
B_1
F
M_1
K_2
M_2
F-I
$\frac{1}{L_1}$
ϕ_1
ϕ_2
$\frac{1}{R_1}$
I
C_1
$\frac{1}{L_1}$
C_2

I	F	V
C ←	M →	L
1/R	B →	R
1/L ←	K →	1/C
φ	x	q

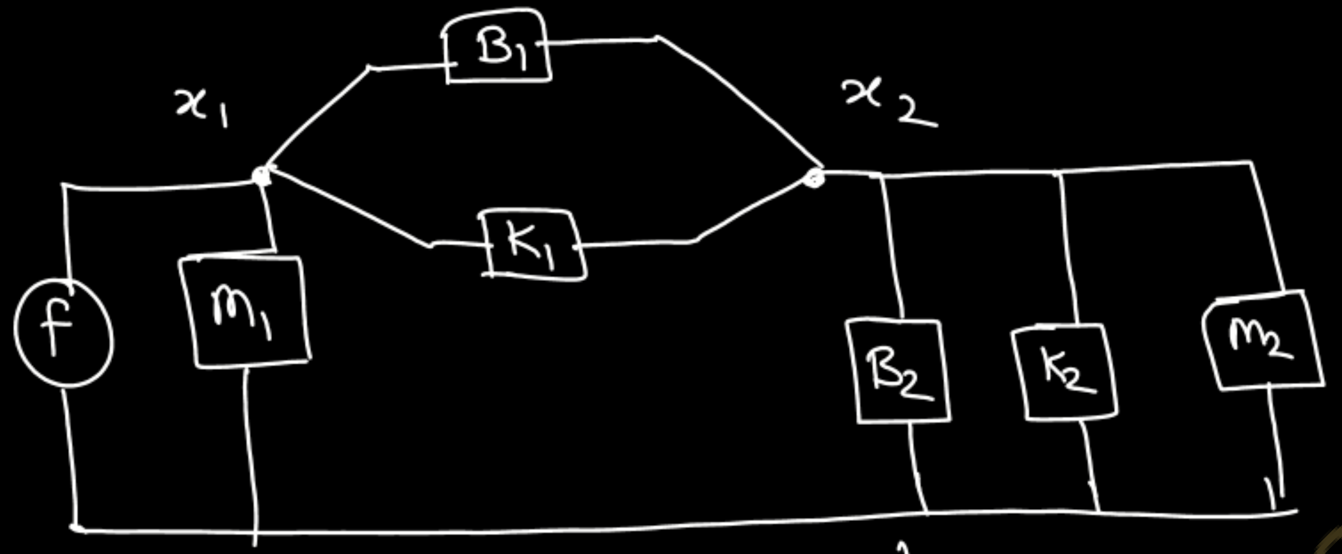
Ex. 2 Write down the differential equation governing the mechanical translation system and Draw analogous electrical System using Force current Analogy.

Sol^n from FBD – 1

$$F = M_1\frac{d^2x_1}{dt^2} + B_1\frac{d}{dt}(x_1-x_2) + K_1(x_1-x_2) \quad \text{—} ①$$

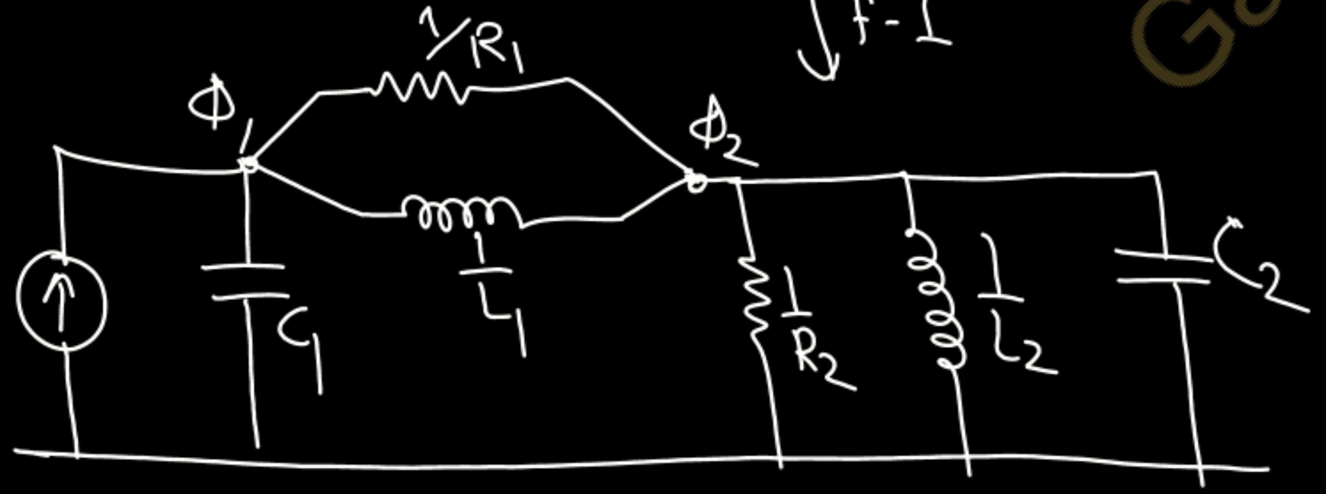
from F B-D-2

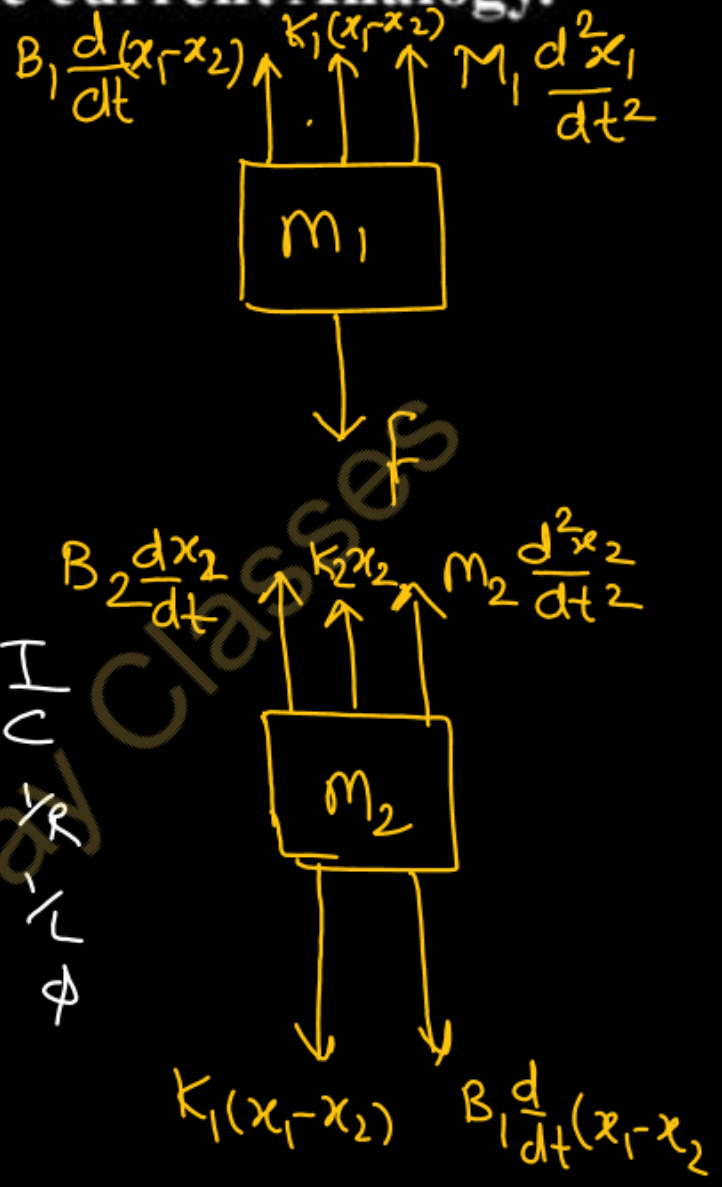
$$B_1\frac{d}{dt}(x_1-x_2)+K_1(x_1-x_2) = M_2\frac{d^2x_2}{dt^2} + K_2x_2 + B_2\frac{dx_2}{dt} \quad \text{—} ②$$

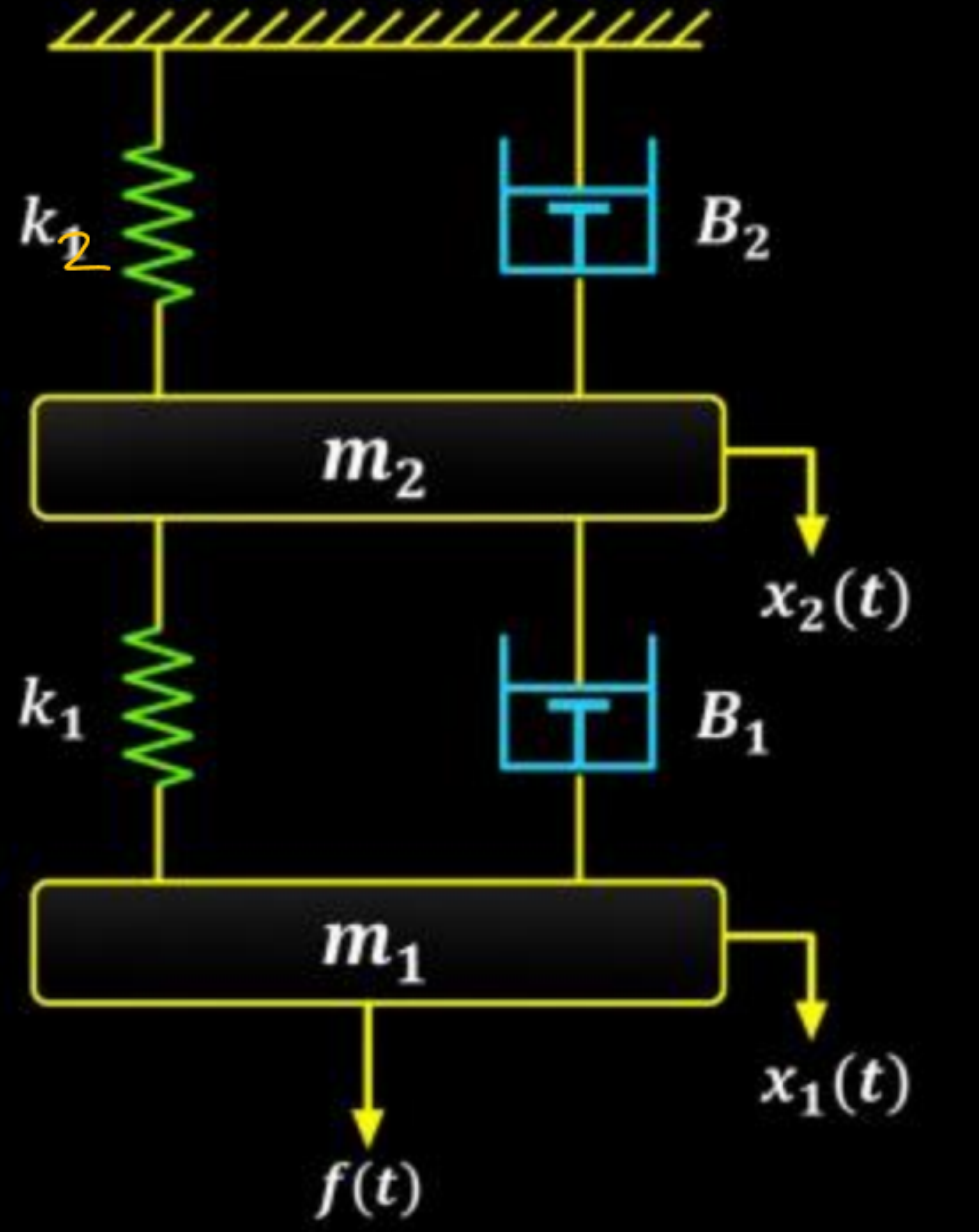

x_1
B_1
x_2
K_1
f
m_1
B_2
K_2
m_2

f	I
m	C
B	1/R
K	1/L
x	φ

$\downarrow$ f-I


ϕ_1
$1/R_1$
ϕ_2
$\frac{1}{L_1}$
C_1
$\frac{1}{R_2}$
$\frac{1}{L_2}$
C_2

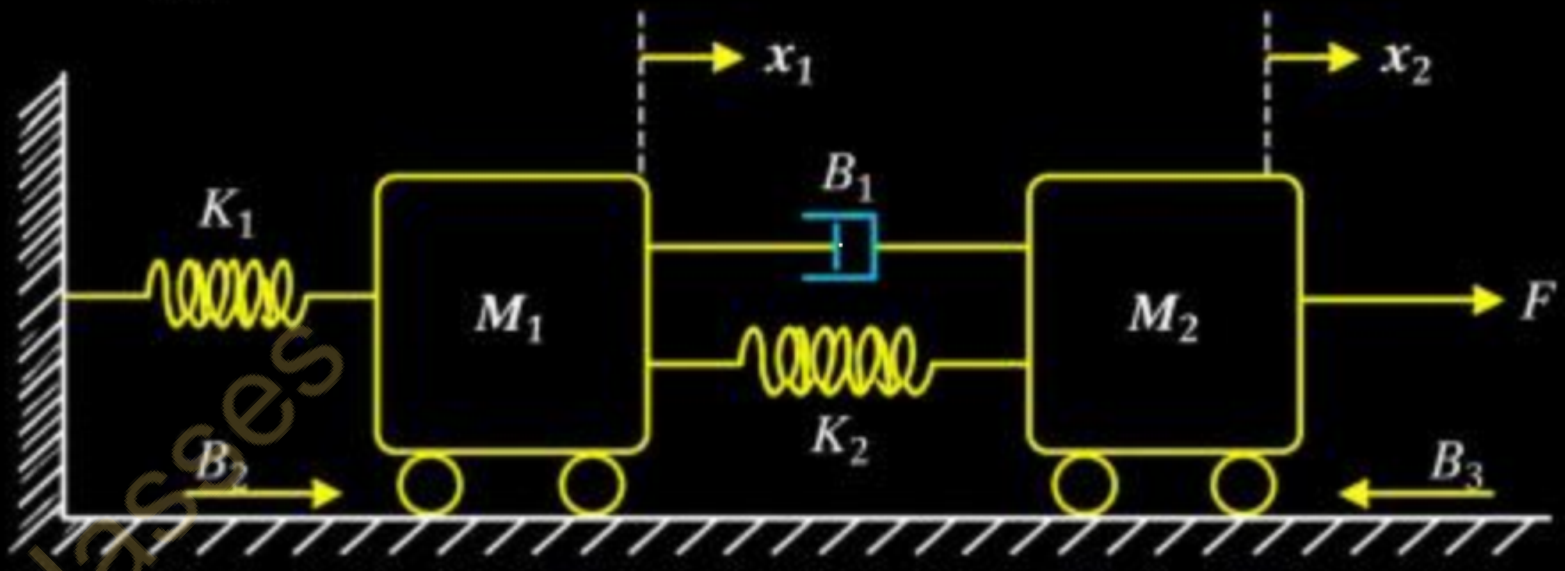

$B_1\frac{d}{dt}(x_1-x_2)$
$K_1(x_1-x_2)$
$M_1\frac{d^2x_1}{dt^2}$
m_1
F
$B_2\frac{dx_2}{dt}$
K_2x_2
$m_2\frac{d^2x_2}{dt^2}$
m_2
$K_1(x_1-x_2)$
$B_1\frac{d}{dt}(x_1-x_2$


k_2
B_2
m_2
$x_2(t)$
k_1
B_1
m_1
$x_1(t)$
$f(t)$

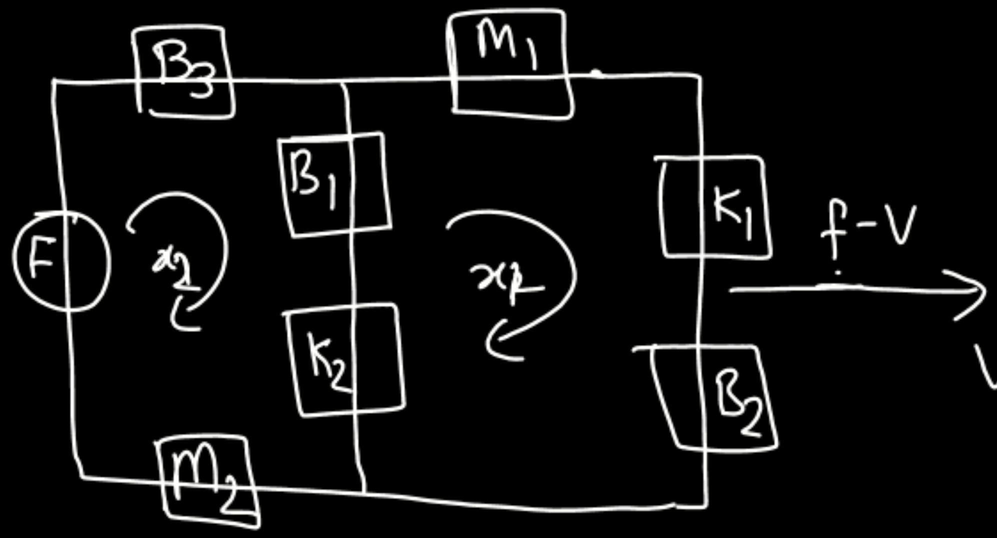
Ex.31 Write down the differential equation governing the mechanical translation system Draw analogous electrical System using Force voltage Analogy.

$$F = m_2\frac{d^2x_2}{dt^2} + B_1\frac{d(x_2-x_1)}{dt} + K_2(x_2-x_1) + B_3\frac{dx_2}{dt} \quad \text{— ①}$$

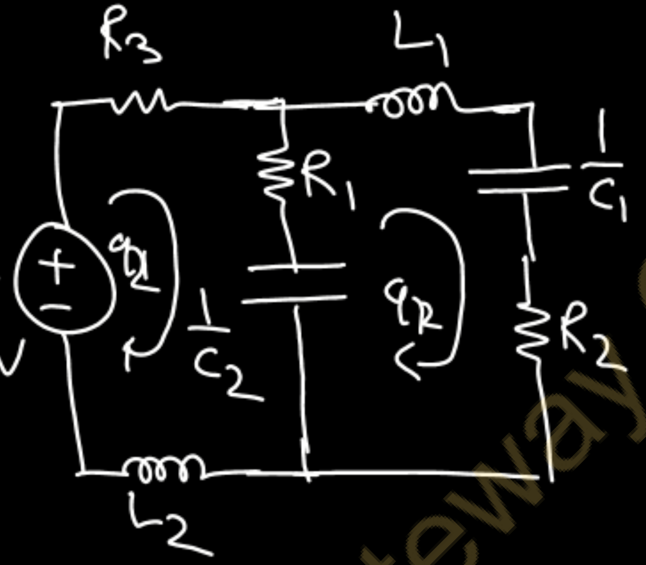
$$B_1\frac{d(x_2-x_1)}{dt} + K_2(x_2-x_1) = M_1\frac{d^2x_1}{dt^2} + K_1x_1 + B_2\frac{dx_1}{dt} \quad \text{— ②}$$



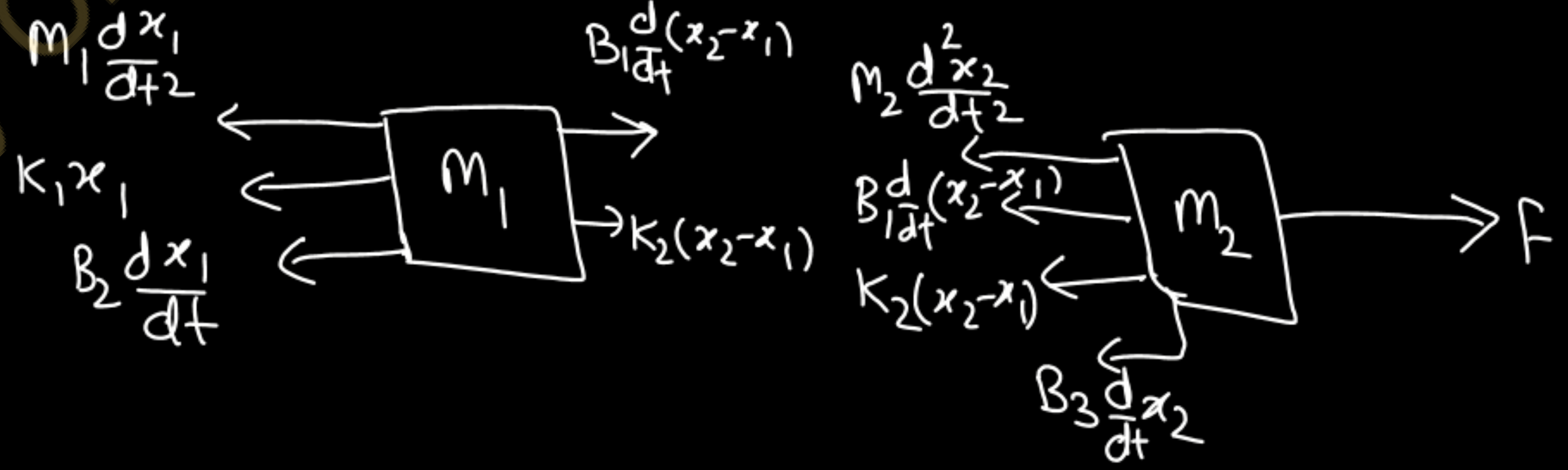
x_1
x_2
K_1
B_1
M_1
M_2
F
K_2
B_2
B_3



B3
M1
B1
K1
F
x2
x1
K2
B2
M2
f-V



R3
L1
R1
1/C1
+
−
V
q1
q2
1/C2
R2
L2



$M_1\frac{dx_1}{dt^2}$
K_1x_1
$B_2\frac{dx_1}{dt}$
M_1
$B_1\frac{d}{dt}(x_2-x_1)$
$K_2(x_2-x_1)$
$M_2\frac{d^2x_2}{dt^2}$
$B_1\frac{d}{dt}(x_2-x_1)$
$K_2(x_2-x_1)$
$B_3\frac{d}{dt}x_2$
M_2
F

f	V
M	L
B	R
K	1/C
x	q

Ex.4 Determine the transfer function X(s)/V(s)

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Step-1 Electrical Equation

Applying KVL,

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + e_b(t)$$

Where, Back emf $e_b(t) = K_b\frac{dx(t)}{dt}$

Taking Laplace Transform,

$$V(s) = RI(s) + LsI(s) + K_bsX(s)$$

or

$$V(s) = (R + Ls)I(s) + K_bsX(s)$$

Step-2 Mechanical Equation

Applied electromagnetic force

$$F(t) = K_fi(t)$$

Applying D Alembert principle,

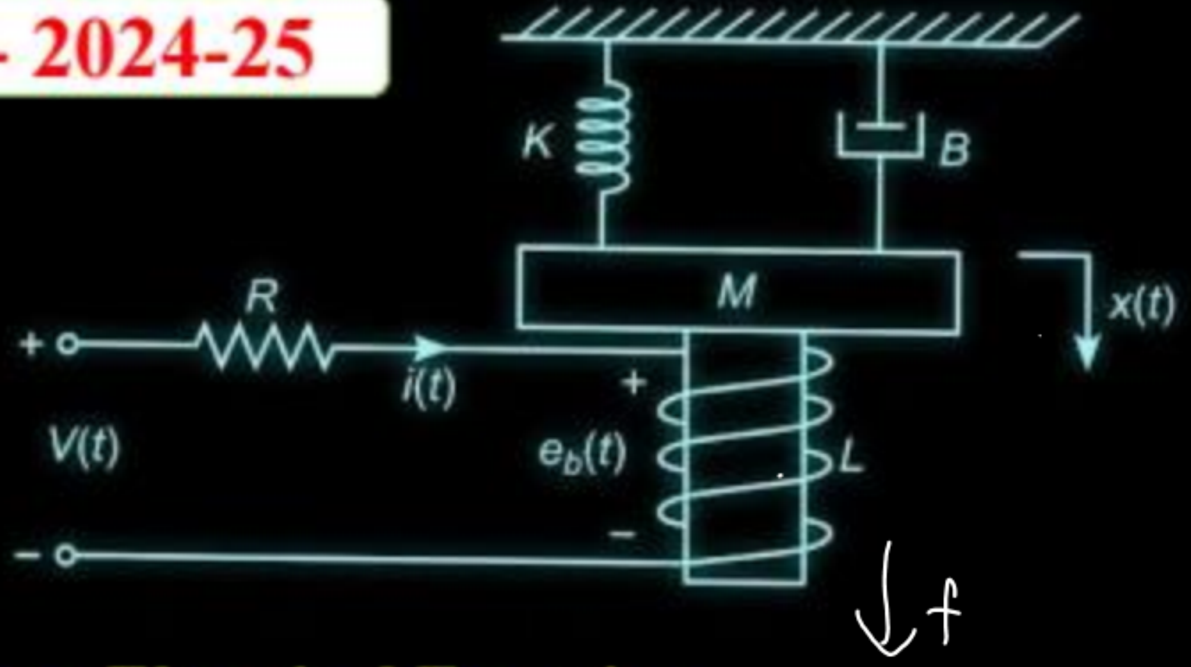
$$K_fi(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

Taking Laplace Transform,

$$K_fI(s) = (Ms^2 + Bs + K)X(s)$$

Hence,

$$I(s) = \frac{(Ms^2+Bs+K)}{K_f}X(s)$$



Step-3 Substitute into Electrical Equation

Substitute $I(s)$ into $V(s) = (R + Ls)I(s) + K_bsX(s)$

$$V(s) = (R + Ls)\frac{(Ms^2 + Bs + K)}{K_f}X(s) + K_bsX(s)$$

Factor out $X(s)$,

$$V(s) = X(s)\left[\frac{(R + Ls)(Ms^2 + Bs + K)}{K_f} + K_bs\right]$$

Multiply numerator and denominator by K_f,

$$V(s) = \frac{X(s)}{K_f}[(R + Ls)(Ms^2 + Bs + K) + K_fK_bs]$$

Therefore,

$$\boxed{\frac{X(s)}{V(s)} = \frac{K_f}{(R + Ls)(Ms^2 + Bs + K) + K_fK_bs}}$$

AKTU

B.Tech Sem-III

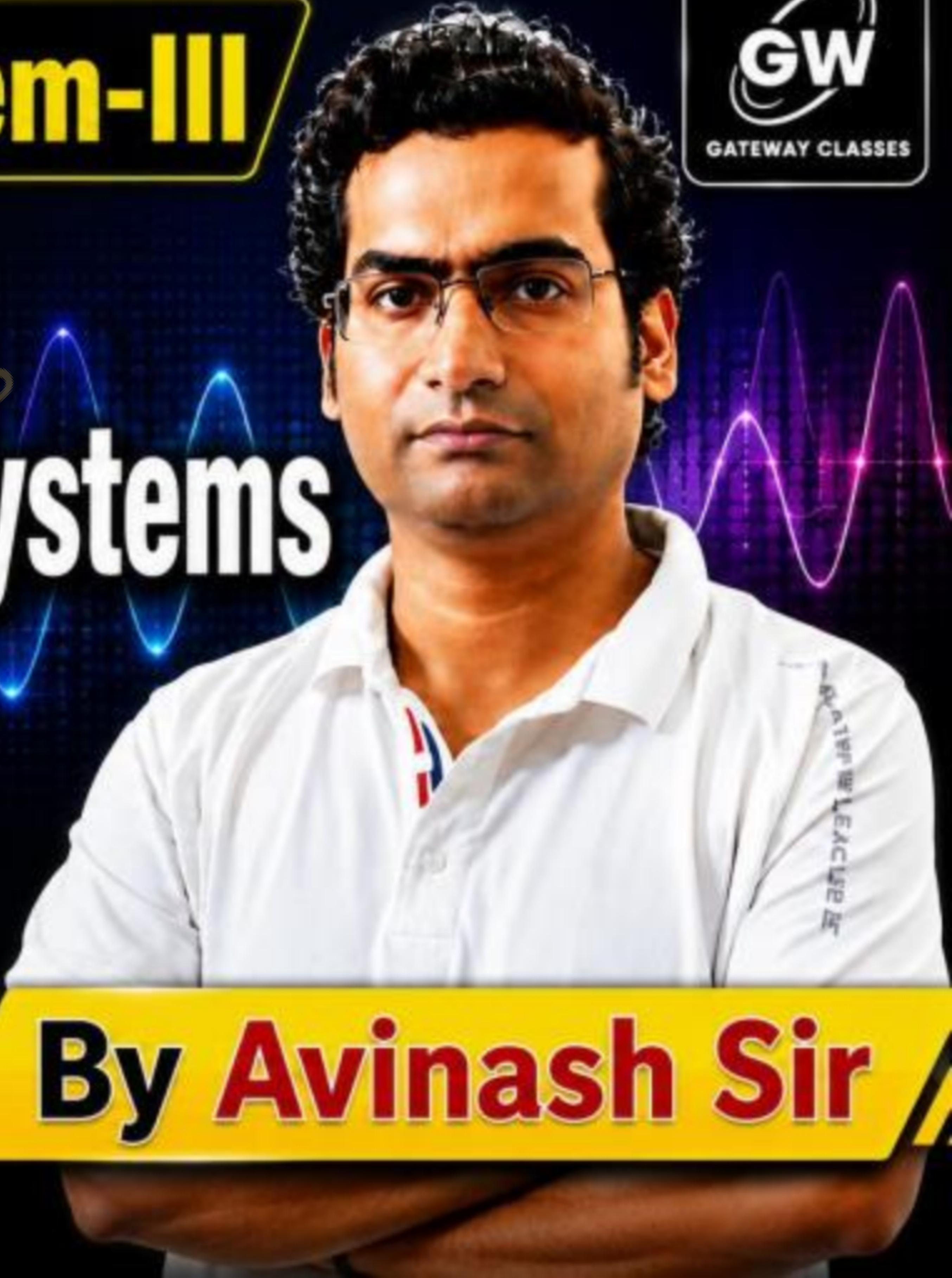
EE and EEE Branch

BSS: Basic Signals & Systems

Unit-1 : Lec-4

Topics to be covered

- Analysis of First Order System
- Analysis of Second Order System
- PYQs



By Avinash Sir

First Order System by Classical Method

A **first-order system** is a system whose highest derivative is of **first order**. Its differential equation is

$$\tau \frac{dy(t)}{dt} + y(t) = x(t)$$

where

- $x(t)$= Input ,$y(t)$= Output ,τ= Time Constant (seconds)

For **unit step input**,

$$x(t) = 1$$

$$\left\{ \begin{array}{l} \text{Impulse I/P} \\ x(t) = 0 \;, t > 0 \end{array} \right\}$$

So,

$$\tau \frac{dy(t)}{dt} + y(t) = 1$$

or

$$\frac{dy}{dt} + \frac{1}{\tau} y = \frac{1}{\tau}$$

Step 1: Complementary Function

First solve homogeneous equation:

$$\frac{dy}{dt} + \frac{1}{\tau} y = 0$$

$$\frac{dy}{dt} = -\frac{1}{\tau} y$$

$$\frac{dy}{y} = -\frac{dt}{\tau}$$

Integrating,

$$\ln y = -\frac{t}{\tau} + C$$

$$y_c = Ce^{-t/\tau}$$

Step 2: Particular Integral

For constant input 1, assume

$$y_p = A$$

Then,

$$\frac{dy_p}{dt} = 0$$

Substitute in equation:

$$\tau(0) + A = 1$$

$$A = 1$$

So,

$$y_p = 1$$

First Order System by Classical Method

Step 3: Complete Solution

$$y(t) = y_c + y_p$$

$$y(t) = Ce^{-t/\tau} + 1$$

Step 4: Apply Initial Condition

For initially relaxed system,

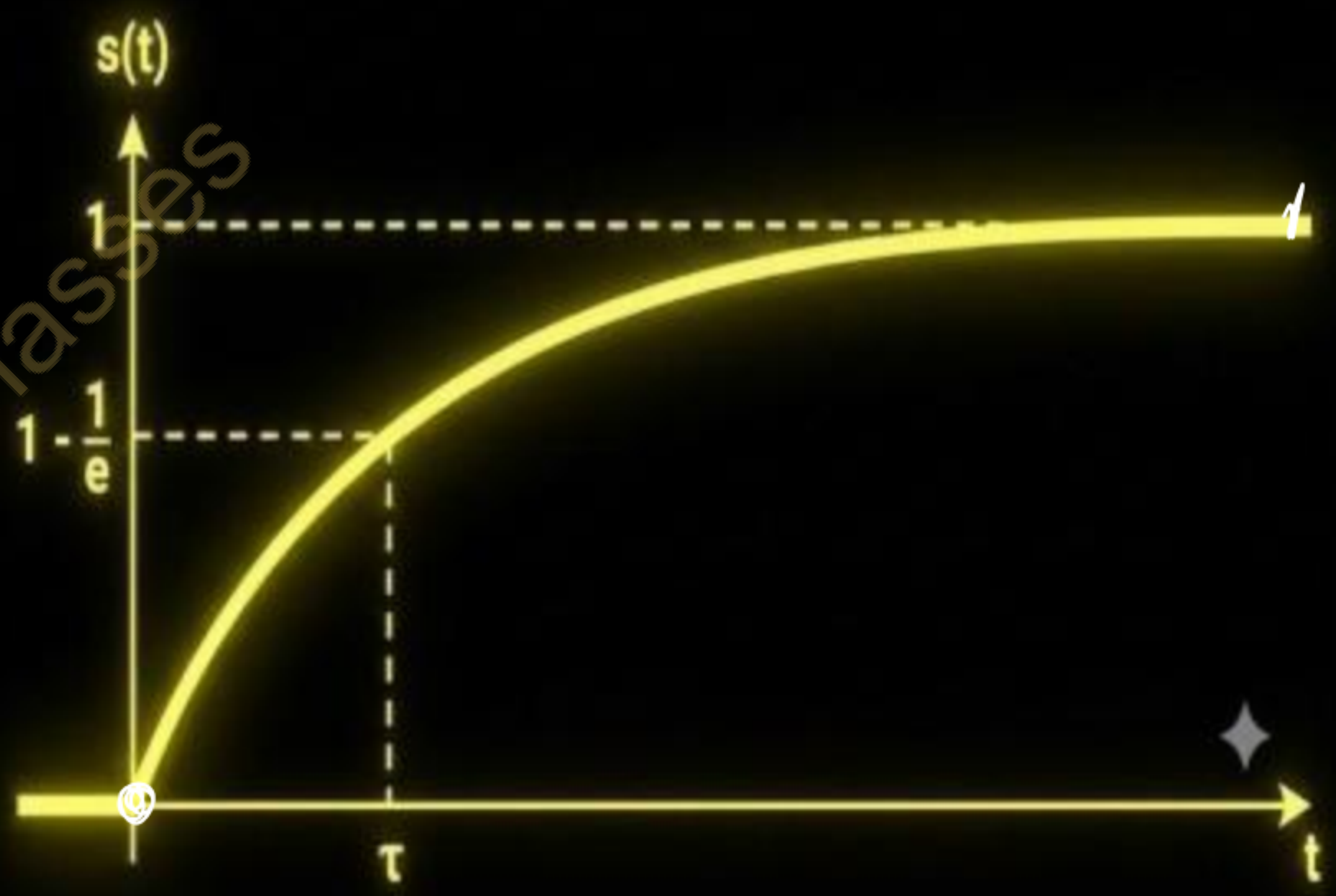
$$y(0) = 0$$

$$0 = Ce^{0} + 1$$

$$C = -1$$

Final Step Response

$$\boxed{y(t) = 1 - e^{-t/\tau}}$$



At $t = 0$,

$$y(0) = 0$$

At $t = \infty$,

$$y(\infty) = 1$$

At $t = \tau$,

$$y(\tau) = 1 - e^{-1}$$

$$\boxed{y(\tau) = 0.632}$$

So after one time constant, output reaches **63.2%** of final value.

Second Order System by Classical Method

Standard second-order differential equation:

$$\frac{d^2y}{dt^2}+2\zeta\omega_n\frac{dy}{dt}+\omega_n^2y=\omega_n^2x(t)$$

For **unit step input**,

$$x(t)=1$$

So,

$$\frac{d^2y}{dt^2}+2\zeta\omega_n\frac{dy}{dt}+\omega_n^2y=\omega_n^2 \rightarrow \text{constant}$$

1. Complementary Function

Homogeneous equation:

$$\frac{d^2y}{dt^2}+2\zeta\omega_n\frac{dy}{dt}+\omega_n^2y=0$$

Auxiliary equation:

$$m^2+2\zeta\omega_nm+\omega_n^2=0$$

Roots:

$$m=-\zeta\omega_n\pm\omega_n\sqrt{\zeta^2-1}$$

2. Particular Integral

Since RHS is constant ω_n^2, assume

$$y_p=A$$

Then,

$$\frac{dy_p}{dt}=0,\frac{d^2y_p}{dt^2}=0$$

Substitute:

$$\omega_n^2A=\omega_n^2$$
$$A=1$$

So,

$$\boxed{y_p=1}$$

Case-1: Underdamped System

When,

$$0<\zeta<1$$

Roots are complex:

$$m=-\zeta\omega_n\pm \underline{j\omega_d}$$

where

$$\omega_d=\omega_n\sqrt{1-\zeta^2}$$

Second Order System by Classical Method

Complementary function:

$$y_c = e^{-\zeta\omega_n t}(C_1 \cos\omega_d t + C_2 \sin\omega_d t)$$

Complete solution:

$$y(t) = \underbrace{1}_{y_p} + e^{-\zeta\omega_n t}(\underline{C_1} \cos\omega_d t + \underline{C_2} \sin\omega_d t)$$

at $t=0$

For initially relaxed system,

$$y(0) = 0, \dot{y}(0) = 0$$

Using these conditions: $C_1 = -1$

and

$$C_2 = -\frac{\zeta}{\sqrt{1-\zeta^2}}$$

Hence,

$$\boxed{y(t) = 1 - e^{-\zeta\omega_n t}\left[\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right]}$$

or

$$\boxed{y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)}$$

where

$$\phi = \cos^{-1}\zeta$$

Case-2: Critically Damped System

When,

$$\zeta = 1$$

Auxiliary equation:

$$m^2 + 2\omega_n m + \omega_n^2 = 0$$
$$(m + \omega_n)^2 = 0$$

Repeated roots:

$$m = -\omega_n,\ -\omega_n$$

Complementary function:

$$y_c = (C_1 + C_2 t)e^{-\omega_n t}$$

Complete solution:

$$y(t) = 1 + (C_1 + C_2 t)e^{-\omega_n t}$$

Using

at $t=0$

$$y(0) = 0, \dot{y}(0) = 0$$

we get

$$C_1 = -1$$
$$C_2 = -\omega_n$$

Therefore,

$$\boxed{y(t) = 1 - (1 + \omega_n t)e^{-\omega_n t}}$$

Case-3: Overdamped System

When, $\zeta > 1$

Roots are real and unequal:

$$m_1 = -\omega_n\left(\zeta - \sqrt{\zeta^2 - 1}\right)$$
$$m_2 = -\omega_n\left(\zeta + \sqrt{\zeta^2 - 1}\right)$$

Complete solution:

$$y(t) = 1 + C_1 e^{m_1 t} + C_2 e^{m_2 t}$$

Using initially relaxed condition, at $t = 0$

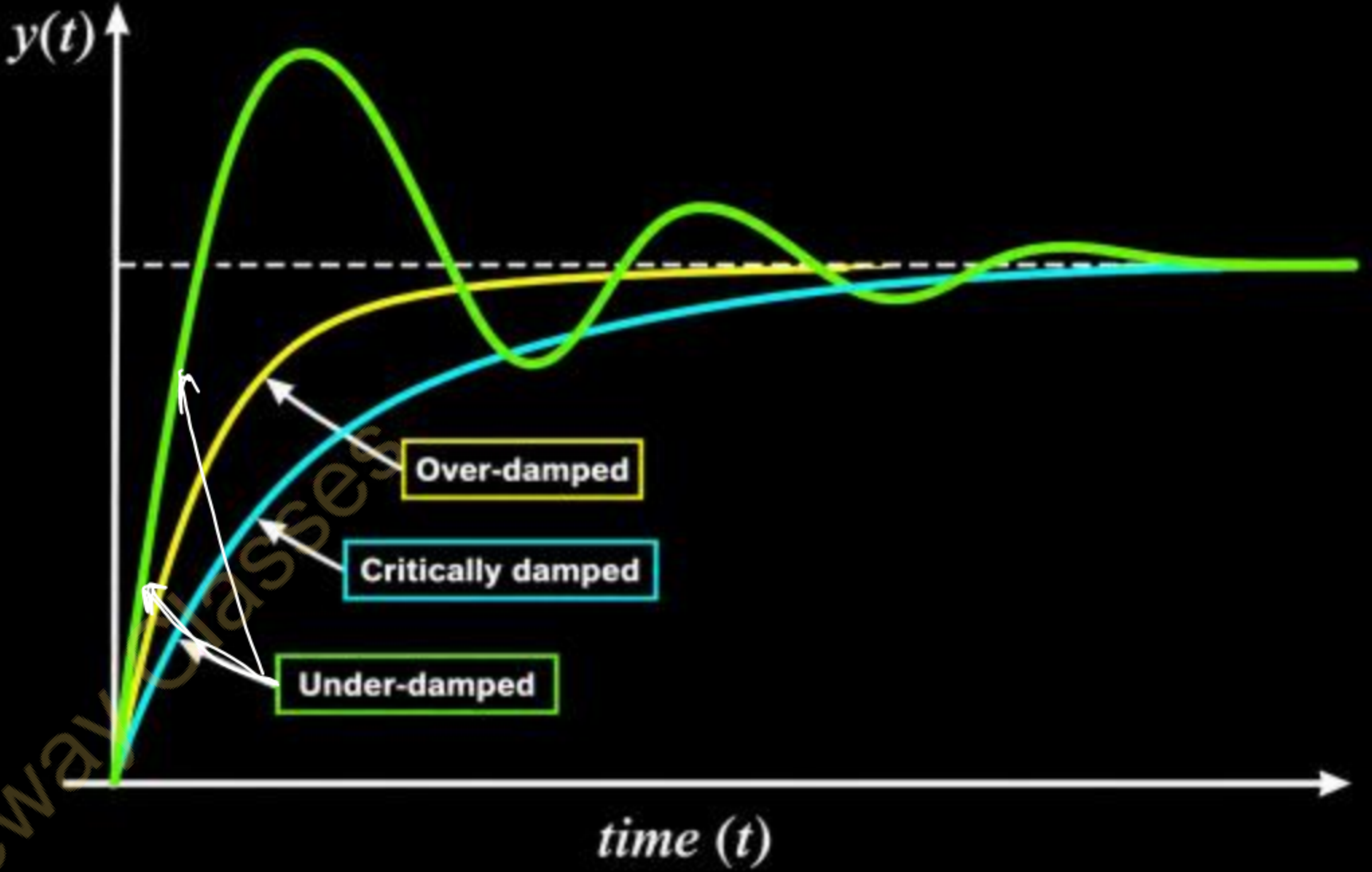
$$y(0) = 0, \dot{y}(0) = 0$$

Final result:

$$\boxed{y(t) = 1 + \frac{m_2 e^{m_1 t} - m_1 e^{m_2 t}}{m_1 - m_2}}$$

where

$$m_1 = -\omega_n\left(\zeta - \sqrt{\zeta^2 - 1}\right)$$
$$m_2 = -\omega_n\left(\zeta + \sqrt{\zeta^2 - 1}\right)$$



Case-4: Undamped System

When, $\zeta = 0$

Equation becomes: $\frac{d^2y}{dt^2} + \omega_n^2 y = \omega_n^2$

Roots:

$$m = \pm j\omega_n$$

Step response:

$$\boxed{y(t) = 1 - \cos\omega_n t}$$

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